Grand Unification Scale Primordial Black Holes: Consequences and Constraints

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A population of very light primordial black holes which evaporate before nucleosynthesis begins is unconstrained unless the decaying black holes leave stable relics. We show that gravitons Hawking radiated from these black holes would source a substantial stochastic background of high frequency gravititational waves (10^{12} Hz or more) in the present Universe. These black holes may lead to a transient period of matter-dominated expansion. In this case the primordial Universe could be temporarily dominated by large clusters of ''Hawking stars'' and the resulting gravitational wave spectrum is independent of the initial number density of primordial black holes.

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Primordial black holes (PBH) produced immediately after the big bang [1,2] can decay via the emission of Hawking radiation [3,4]. The initial PBH population is determined by the primordial perturbation spectrum. Bounds on the PBH population constrain inflation and other early Universe scenarios, which generate this spectrum [5–8]. These constraints follow from the lack of evidence for the present-day existence of PBH, and because decaying black holes disrupt nucleosynthesis, recombination, and reionization [9]. At formation, a PBH must be smaller than the Hubble horizon, and the amount of material inside the Hubble horizon—and the maximal mass and lifetime of a PBH—*increases* as the Universe expands. Very light PBH decay completely before nucleosynthesis, and are consequently unconstrained. A PBH radiates any and all particles whose rest mass is substantially less than its current temperature, including gravitons. If the PBH radiate massive, long-lived particles one obtains tight bounds on their initial population [10–14], but these limits are contingent upon assumptions about particle physics and quantum gravity, and other radiated particles will reach thermal equilibrium, erasing any memory of their origin. However, gravitons emitted as the black hole decays cannot equilibrate and will always survive until the present day, producing a stochastic background of gravitational waves. Further, for some parameter choices the early Universe has a transient matter-dominated phase, during which large clusters of PBH can form. In this case the resulting gravitational wave spectrum is independent of the initial fraction of black holes.

The mass fraction of PBH is denoted $\Omega_{\rm BH}$. Initially $\Omega_{\rm BH} = \beta$, $0 < \beta < 1$. We assume that the remaining matter consists of radiation. PBH form if $\delta\rho/\rho \gtrsim 10^{-2}$ at very short scales, and a number of inflationary models have this property ([15,16], and references therein). Beyond this threshold β rapidly approaches unity. An appreciable gravitational wave background is generated even if β is very small, so given a model which predicts the existence of *any* small PBH, the signal discussed here is generic.

For simplicity, we assume a PBH population whose mass is equal to the energy contained inside the Hubble volume at the instant they collapse. Recalling that $H^2 = 8\pi\rho/3M_p^2$, and defining $\rho = E_{\rm init}^4$,

$$M_{\rm BH} = \sqrt{\frac{3}{32\pi}}\frac{M_p^3}{E_{\rm init}^2} \tag{1}$$

which is the mass contained inside a sphere of radius $1/H$. Including graybody factors, Γ_{sl}, a Schwarzschild black hole emits (massless) particles with momentum k, reducing its total energy as

$$\frac{dE}{dtdk} = -\frac{M_{\rm BH}^2}{2\pi M_p^4}k\sum_{s,l}\frac{(2l+1)h(s)\Gamma_{sl}(kM_{\rm BH}/M_p^2)}{\exp(8\pi M_{\rm BH}k/M_p^2)\pm 1} \tag{2}$$

$$= -\frac{2g}{\pi}\frac{M_{\rm BH}^2}{M_p^4}\frac{k^3}{e^{k/T}-1}, \tag{3}$$

$$T = \frac{M_p^2}{8\pi M_{\rm BH}}, \tag{4}$$

where $h(s)$ counts the helicity or polarization states of a particle with spin s [17,18]. The second line is the pure blackbody expression and g is the *effective* number of (bosonic) degrees of freedom, after graybody corrections. These suppress emission at larger s so g depends on both the mix of spin states and total number of light degrees of freedom. For each state with $s = 0, 1/2, 1, 2$, the contribution to g is 7.18, 3.95, 1.62, 0.18 so graviton emission is an order of magnitude below a naive mode-counting estimate. We ignore angular momentum, which enhances graviton emission [19]. The physical wave number k is $\tilde{k}/a(t)$, where $\tilde{k}$ is the comoving wave number and $a(t)$ is the scale factor. Integrating,

$$\frac{dM_{\rm BH}}{dt} = -\frac{g}{30\,720\pi}\frac{M_p^4}{M_{\rm BH}^2}, \tag{5}$$

from which we can compute the lifetime

$$\tau = \frac{10\,240\pi}{g}\frac{M_{\rm BH}^3}{M_p^4} = \frac{240}{g}\sqrt{\frac{3}{2\pi}}\frac{M_p^5}{E_{\rm init}^6}. \qquad (6)$$

An upper bound on $E_{\rm init}$ comes from the inflationary energy scale, which is constrained by the nondetection of a primordial gravitational wave background in the cosmic microwave background (CMB), which we (generously) take to be $\sim 10^{16}$ GeV. At the lower end we are interested in black holes which decay prior to nucleosynthesis with time to spare for thermalization, so we need $\tau \lesssim 100$ s. With $E_{\rm init} = 10^{12}$ GeV, $\tau \approx 29/g$ s, and the initial temperature is 18.8 TeV. Standard model states alone give $g \sim \mathcal{O}(10^2)$ and in what follows we (conservatively) assume $g \geq 10^3$. Lowering $E_{\rm init}$ slightly ensures that the PBH will survive through nucleosynthesis, so we assume $E_{\rm init} \geq 10^{12}$ GeV.

Gravitational wave background.—Denoting the number density of PBH by $n(t)$, the energy density $\rho_{\rm BH} = n(t)M_{\rm BH}(t)$. We thus solve [20]

$$\frac{d\rho_{\rm BH}}{dt} = \dot{n}(t)M_{\rm BH} + n(t)\dot{M}_{\rm BH}, \qquad (7)$$

$$= -3\frac{\dot{a}}{a}\rho_{\rm BH} + \rho_{\rm BH}\frac{\dot{M}_{\rm BH}}{M_{\rm BH}}, \qquad (8)$$

$$\frac{d\rho_{\rm rad}}{dt} = -4\frac{\dot{a}}{a}\rho_{\rm rad} - \rho_{\rm BH}\frac{\dot{M}_{\rm BH}}{M_{\rm BH}}, \qquad (9)$$

$$\frac{\dot{a}}{a} = \left[\frac{8\pi}{3M_p^2}(\rho_{\rm BH} + \rho_{\rm rad})\right]^{1/2} \qquad (10)$$

along with Eq. (2). One obtains $\Omega_{\rm gw}$ by an appropriate rescaling. We finally compute the present-day spectral energy density of gravitational radiation [23,24],

$$\Omega_{\rm gw}(f) = \frac{1}{\rho}\frac{d\rho_{\rm gw}}{d\ln f}, \qquad (11)$$

where ρ is the overall density and $\rho_{\rm gw}$ is the energy density in gravitational waves. [This quantity depends weakly on $g_\star$, the number of degrees of freedom after the Universe rethermalizes. This differs from the g that fixes τ, as a decaying PBH is much hotter than the surrounding Universe. We take $g_\star = 200$, and plot $\Omega_{\rm gw}(f)h^2$, where h is the dimensionless Hubble parameter].

Figure 1 shows $\Omega_{\rm gw}(f)h^2$ as a function of $E_{\rm init}$. The gravitational wave power is substantial, and at very high frequencies. Roughly speaking, the temperature of the Universe scales as $1/a(t)$. A decaying black hole is much hotter than the surrounding Universe, but the emitted gravitational waves are redshifted by the same factor as other radiation. Consequently, these gravitational waves have a higher frequency than the present-day CMB. Lowering $E_{\rm init}$ increases the PBH lifetime, enhancing this discrepancy and pushing the gravitational wave signal to higher frequencies. The "dip" at very high frequencies arises because these quanta can only be sourced by a small black hole, and are produced in smaller numbers. Conversely, increasing g reduces the fraction of emission into gravitational waves, lowering $\Omega_{\rm gw}$. As τ is inversely proportional to g, the gravitational waves are emitted when the Universe is smaller, increasing the subsequent redshift factor of the emitted radiation, lowering their present-day frequency, as seen in Fig. 2.

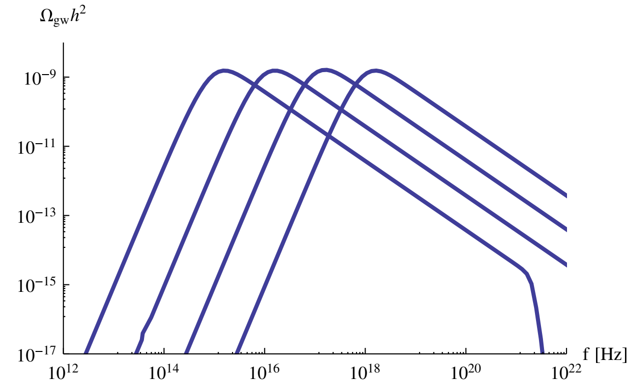

FIG. 1 (color online). $\Omega_{\rm gw}(f)h^2$ with (from left to right) $E_{\rm init} = 10^{15}$, 10^{14}, 10^{13}, and 10^{12} GeV, and $\beta = 0.001$ and $g = 1000$.

Early matter domination.—The primordial Universe is radiation dominated, whereas PBH scale like matter. Initially, $\Omega_{\rm BH} \propto a(t)$ until either $\Omega_{\rm BH} \approx 1$ or the PBH reach the final phase of their evaporation and $\Omega_{\rm rad}$ begins to grow. If the Universe does become PBH dominated, all the radiation in the "late" Universe will have been emitted by PBH, with the "original" radiation making a negligible contribution. In this case $\Omega_{\rm gw} = 0.36/g$ after evaporation and the graviton spectrum is independent of β. If β is very small or g very large, the Universe is always radiation dominated and $\Omega_{\rm gw} < 0.36/g$. Setting $\rho_{\rm rad} \approx E_{\rm init}^4$, a matter-dominated phase takes place if

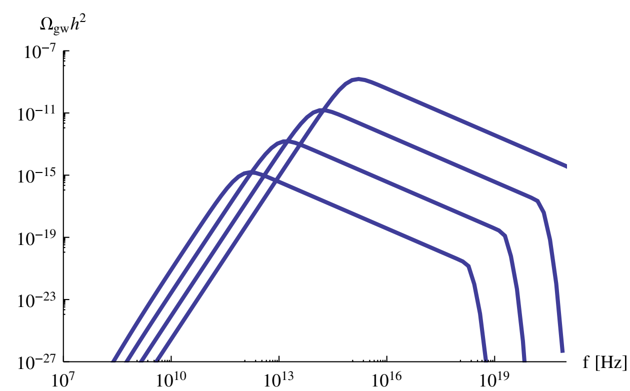

FIG. 2 (color online). $\Omega_{\rm gw}(f)h^2$ with (from top to bottom) $g = 10^3$, 10^5, 10^7, and 10^9. In all cases $\beta = 0.001$ and $E_{\rm init} = 10^{15}$ GeV.

$$\beta \gtrsim \frac{1}{8}\sqrt{\frac{g}{15}}\frac{E_{\rm init}^2}{M_P^2}. \quad (12)$$

Recall that $H^2 = 1/4t^2$ in a radiation dominated universe. If the Universe remains radiation dominated until the PBH have decayed, its grows by $a(\tau + t_{\rm init}) \approx (\tau/t_{\rm init})^{1/2}$ with $a(t_{\rm init}) \equiv 1$. Thus, $\Omega_{\rm gw}(f)$ decreases with β if the above inequality is not satisfied, as shown in Fig. 3. Figure 4 shows the region of parameter space for which a matter-dominated phase occurs, while Fig. 5 shows $\Omega_{\rm BH}$ and $\Omega_{\rm rad}$ for a specific scenario with a lengthy matter-dominated phase.

In a radiation dominated universe, $H(t) \sim 1/a(t)^2$. As always $1/H$ defines the physical Hubble scale while the *comoving* Hubble distance is $a(t)/H_{\rm init}$. The number of PBH per initial Hubble volume is β, so before matter domination, the number of PBH per Hubble volume is $\beta a(t)^3$. This number can be large: in Fig. 5, $\beta = 10^{-8}$, and $a(t) = 10^8$ before PBH domination, so there are 10^{16} PBH within a single Hubble horizon. Perturbations grow in a matter-dominated universe. A mode which is inside the horizon and longer than the Jeans length has amplitude $\delta \propto \eta^2$, $\eta = \int dt/a(t)$ [25]. During matter domination, $\delta \sim a(t)$, and short scales become nonlinear. Moreover, in order to ensure the formation of PBH, the initial amplitude of the perturbations is presumably substantially larger than the canonical 10^{-5} found at astrophysical scales. A PBH dominated phase may thus be accompanied by the growth of nonlinear structure at subhorizon scales, leading to the formation of large clusters of PBH. This situation is reminiscent of the present Universe, with the decaying, clustered PBH playing the role of "Hawking stars."

The possibility that PBH cause a transient matter-dominated phase has been discussed previously (e.g., [21]) and a universe dominated by decaying PBH is in thermal equilibrium and thus a potential site for baryogenesis [21,26,27]. Crucially, the formation of nonlinear overdensities could dramatically enhance the interaction rates *between* black holes [28,29]. If two PBH merge, the resulting black hole lives roughly 8 times longer than the parent objects. If a typical PBH survives until shortly before the onset of nucleosynthesis, a small population of longer lived black holes is potentially troublesome. Since the lifetime of the PBH depends very strongly on the initial energy, we see from Eq. (6) that a factor of 10 in τ can be eliminated by increasing $E_{\rm init}$ by a factor $10^{1/6} \approx 1.5$, and the lower bound on $E_{\rm init}$ will only change substantially if many PBH coalesce into single objects.

To put a crude lower bound on the merger rate, recall that our horizon-mass PBH have a Schwarzschild radius $r_s = 2M/M_p^2$ which is equal to the initial Hubble length, $1/H$. Assume that PBH separated by an initial comoving distance of cr_S will merge, where c is a number of order unity. In a comoving region of radius cr_S, we expect to find $\sim c^3\beta$ PBH, so volumes with N PBH will be $\sim(c^3\beta)^{(N-1)}$ rarer than volumes with just one PBH. Thus, if we reach a matter-dominated phase the fraction of the Universe composed of PBH with mass $NM_{\rm BH}(t_{\rm init})$ is $\sim(c^3\beta)^{(N-1)}$. Unless β is close to unity this initial merger phase will not yield a long-lived population of PBH. However, correlations in the initial distribution of PBH [29] or the formation of large nonlinear clusters of PBH could substantially enhance the merger rate.

Discussion.—We show that light PBH which evaporate before nucleosynthesis lead to a high frequency gravitational wave background. At present, this is of theoretical interest, given that this background is at frequencies far beyond the sensitivity region of LIGO, or proposed space-based interferometers such as LISA, which are the most sensitive gravitational wave experiments currently in development. However, the existence of plausible high frequency backgrounds motivates the development of novel

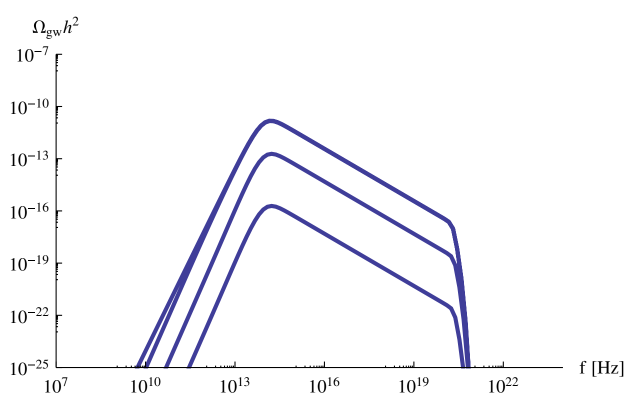

FIG. 3 (color online). $\Omega_{\rm gw}(f)h^2$ with (from top to bottom) $\beta = 10^{-3}$, 10^{-6}, 10^{-9}, and 10^{-12}. The $\beta = 10^{-3}$ and 10^{-6} cases lie on top of each other. In all cases $g = 10^5$ and $E_{\rm init} = 10^{15}$ GeV.

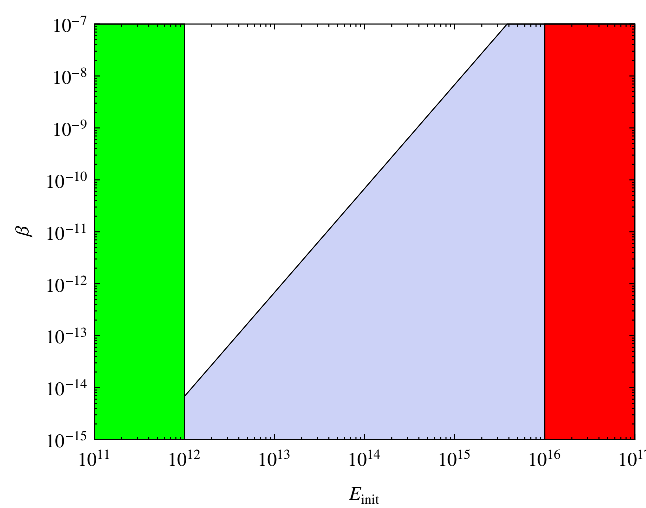

FIG. 4 (color online). The region in the $\{E_{\rm init}, \beta\}$ for which a matter-dominated period is allowed is plotted for $g = 1000$ (white area), along with the generic requirement that $10^{12} < E_{\rm init} < 10^{16}$.

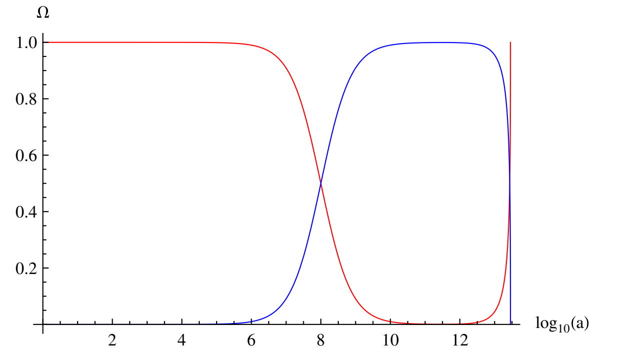

FIG. 5 (color online). Ω_{BH} and Ω_{rad} are plotted for a scenario with $E_{init} = 10^{13}$ GeV, $\beta = 10^{-8}$, and $g = 1000$.

detector technologies. The spectral density of this background is substantial, and may exceed that obtainable from phase transitions or bubble collisions [23,30]. Further, a light PBH population can lead to a temporary period of matter domination before the onset of nucleosynthesis during which clusters of PBH could form, leading to a cold phase during which the primordial Universe is dominated by clusters of Hawking stars.

Gravitational waves generated during preheating or parametric resonance at the end of inflation have recently received considerable attention [23,31,32]. Decaying PBH thus provide a further mechanism by which inflation—if it sources perturbations which lead to the formation of PBH—may generate a high frequency gravitational wave background. Very simple models of inflation do not yield PBH and thus have $\beta \equiv 0$, but current bounds on the running of the spectral index $\alpha = dn_s/d\ln k$ are compatible with PBH production [8]. Further, these bounds are obtained by extrapolating the full inflaton potential from the region traversed as astrophysical perturbations are generated. This is not valid for models where inflation ends abruptly, and these scenarios can lead to substantial PBH production [15], although the simplest models of this form often predict $n_s > 1$, which is in conflict with current data. Consequently, we simply treat β as a free parameter, although it would be computable in any well-specified inflationary scenario. However, note that it need not be large—even if $\beta < 10^{-10}$ one may still have a lengthy matter-dominated phase, provided E_{init} is at the lower end of the allowed range.

The analysis here contains a number of simplifying assumptions. However, these do not affect our basic conclusion that a high frequency gravitational wave background generated by Hawking radiation is the only signature of a quickly decaying PBH population which certainly survives until the present epoch.

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