ORDER OF MAGNITUDE PHYSICS

RICHARD ANANTUA

JING LUAN

JEFFREY FUNG

COURSE LOGISTICS
ORDER OF MAGNITUDE PHYSICS

- Instructors: Richard Anantua, Jing Luan, and Jeffrey Fung
- Contact email: astron9d@gmail.com
- Days and time: MWF 1:30-4:00p at Campbell 121 (with 10 min break most classes)
- Office Hours: by appointment
- Materials:
  - Caltech OOM Notes: http://www.inference.org.uk/sanjoy/oom/book-a4.pdf
  - Sanjoy Mahajan, The Art of Insight in Science and Engineering: Mastering Complexity : https://tinyurl.com/z8cj6gc

COURSE LOGISTICS
ORDER OF MAGNITUDE PHYSICS

- HWs (35%): Weeks 2, 3, 5 and 6.
  - LATE POLICY: Up to 1 week late => 50% off; over 1 week late => 100% off
- Midterm Presentation (15%): Week 4
- Midterm Paper (15%): Week 4
- Final Exam (30%): Wednesday, Week 6 (Last week of class)
- Participation (5%): All day, every day (hopefully)

What is Order of Magnitude Physics?

The art of computing practical answers
to physical problems in a fast and efficient manner
by sacrificing some precision.

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The art of computing practical answers
to physical problems in a fast and efficient manner
by sacrificing some precision.

In other words,

find the best estimate in the simplest way possible.

But Why?

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1) Sometimes that is the best you can do.

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You are an engineer assigned to remove this boulder.

To do your job, you need an estimate of its weight.

How do you do that?

But Why?

2) Sometimes we just want a quick answer.

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You are having a conversation with a colleague. He says, ``I wonder how many astronomers are there.''

Option A: ``Hold my beer. Let me go conduct a full-scale worldwide survey.''

Option B: ``There are about 100 schools granting graduate astronomy degrees. Say 5 graduates per year per school, and an average career of 30 years. There are probably more than 10,000 astronomers in the world.''

But Why?

3) But most importantly, it gives you insight.

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You try to solve: $\int_0^1 e^{\sin\left[(x+1)^4\right]}dx$

After some gymnastics, you find: $\dfrac{5\pi}{3}$

Are you right?

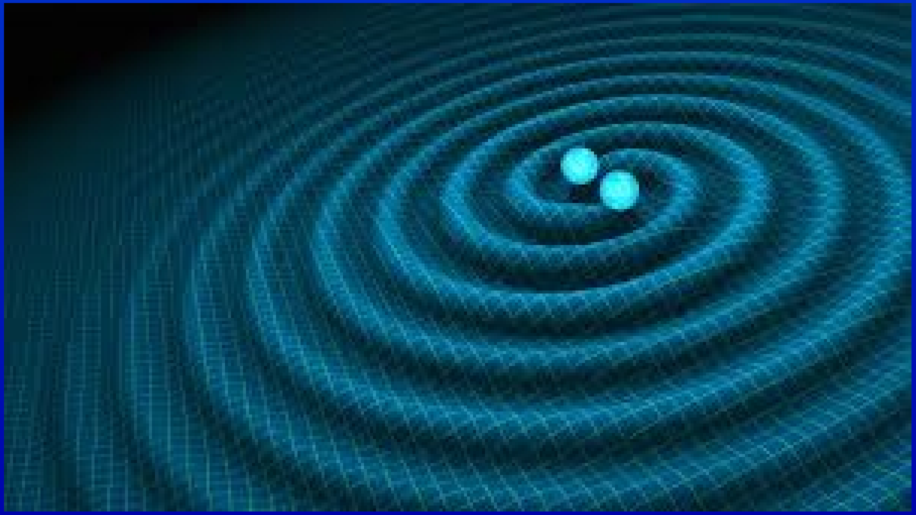
But Why?

3) But most importantly, it gives you insight.

You read an internet article titled:

“Gravitational Waves from Nearby Black Hole Collisions Could Destroy Earth”

Is it fake news?



But Why?

4) … and maybe, write a paper using it?

THE ASTROPHYSICAL JOURNAL

Stellar Winds and Dust Avalanches in the AU Mic Debris Disk

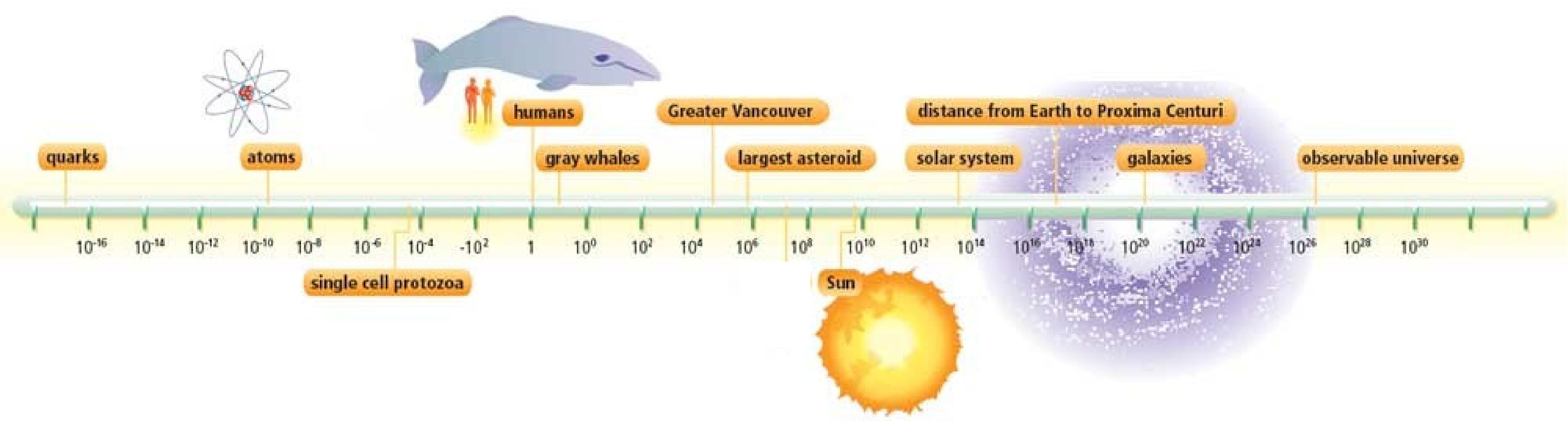
Eugene Chiang[1,2,4] and Jeffrey Fung[1,3,4]

Published 2017 October 5 •

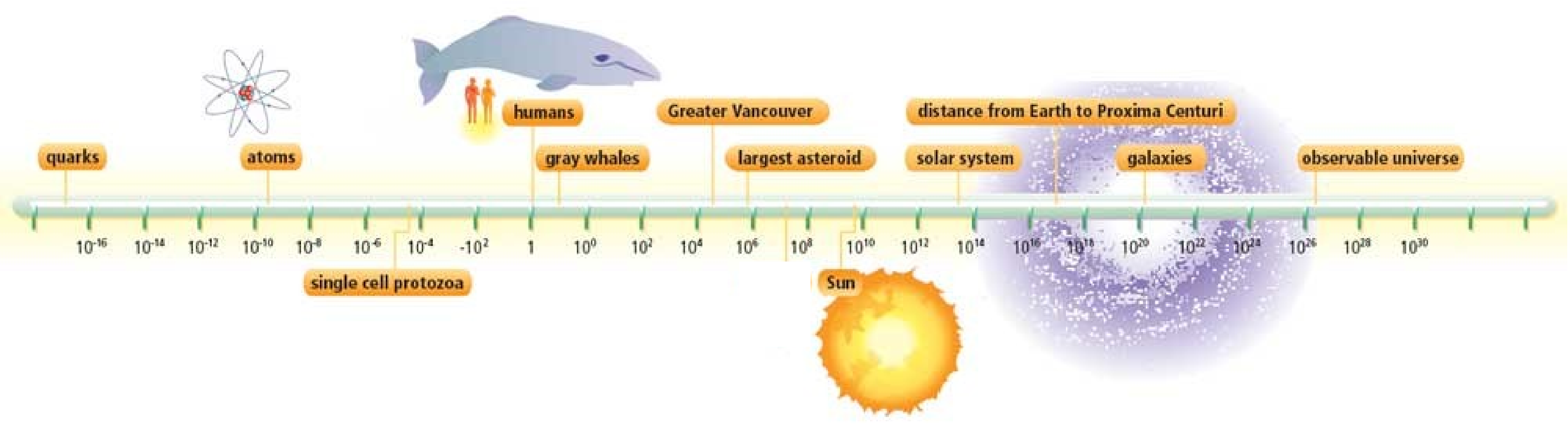
The Astrophysical Journal, Volume 848, Number 1

I am serious!

Order of Magnitude

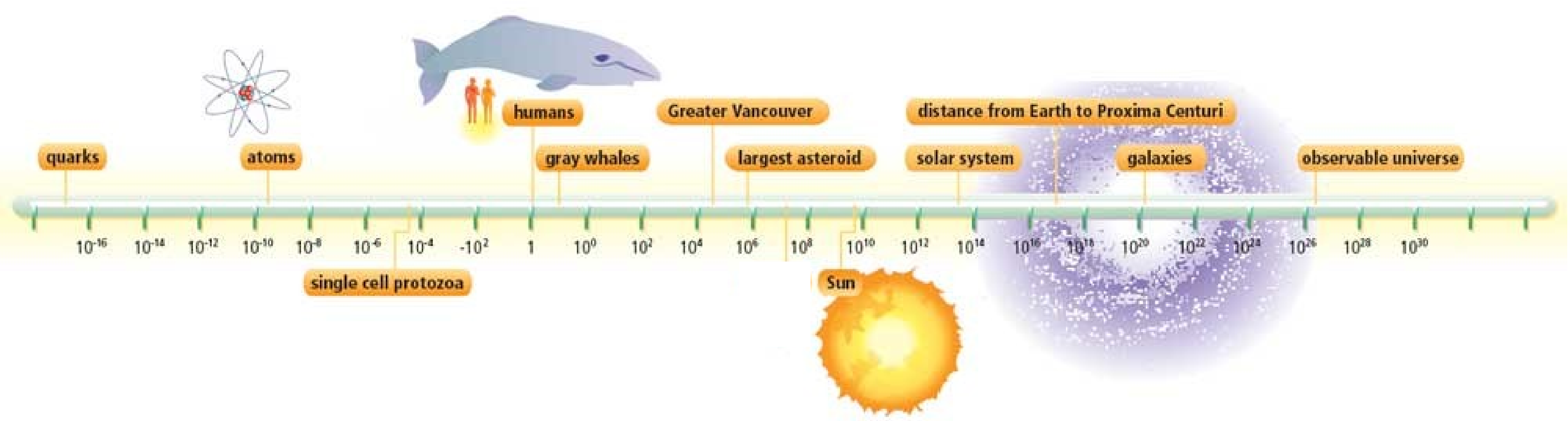


Order of Magnitude



Question: How tall am I?

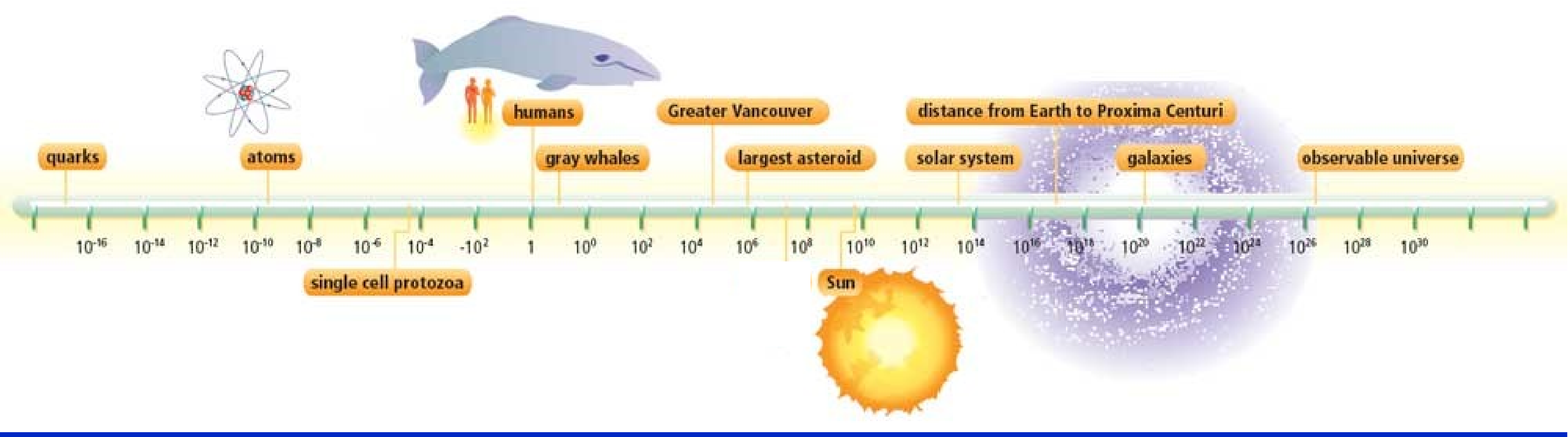
Order of Magnitude



Question: How tall am I?

Answer: 1 m.
(This implies the answer lies between 0.1 to 10m.)

Order of Magnitude



Question: How tall am I?

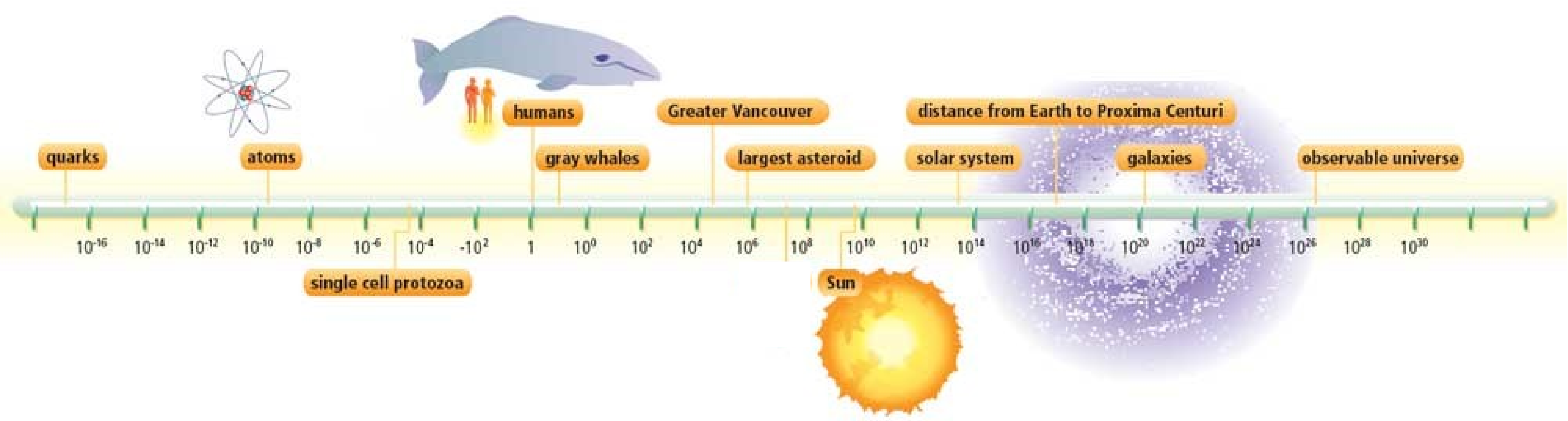
Answer: 1 typical human height (1.6m).

Order of Magnitude

1. No “correct” answers in this class, but there are “better” and “worse” answers.

2. We care about how you get your answers! In fact, we care most about that.

Order of Magnitude



Question: If I make a 30 cm globe to scale, how tall should I make the mountains?

Dimensional Analysis

So what is the mass of this boulder?

Dimensional Analysis

So what is the mass of this boulder?

Typical density of rock ~ 2 g / cm^3

Typical size of a sheep ~ 1m

$$\begin{aligned} mass &= density \times volume \\ &\sim 2\, g/cm^3 \times 2\, m \times 2\, m \times 4\, m \\ &\sim 30\, tonnes \end{aligned}$$

Fermi Problems

Problems that can be answered using rough but quantitative estimates, made famous by Enrico Fermi.

How many piano tuners are there in Chicago?

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~~How many piano tuners are there in Chicago?~~

How many intelligent extraterrestrial civilizations are there in the Milky Way? (aka the Drake equation)

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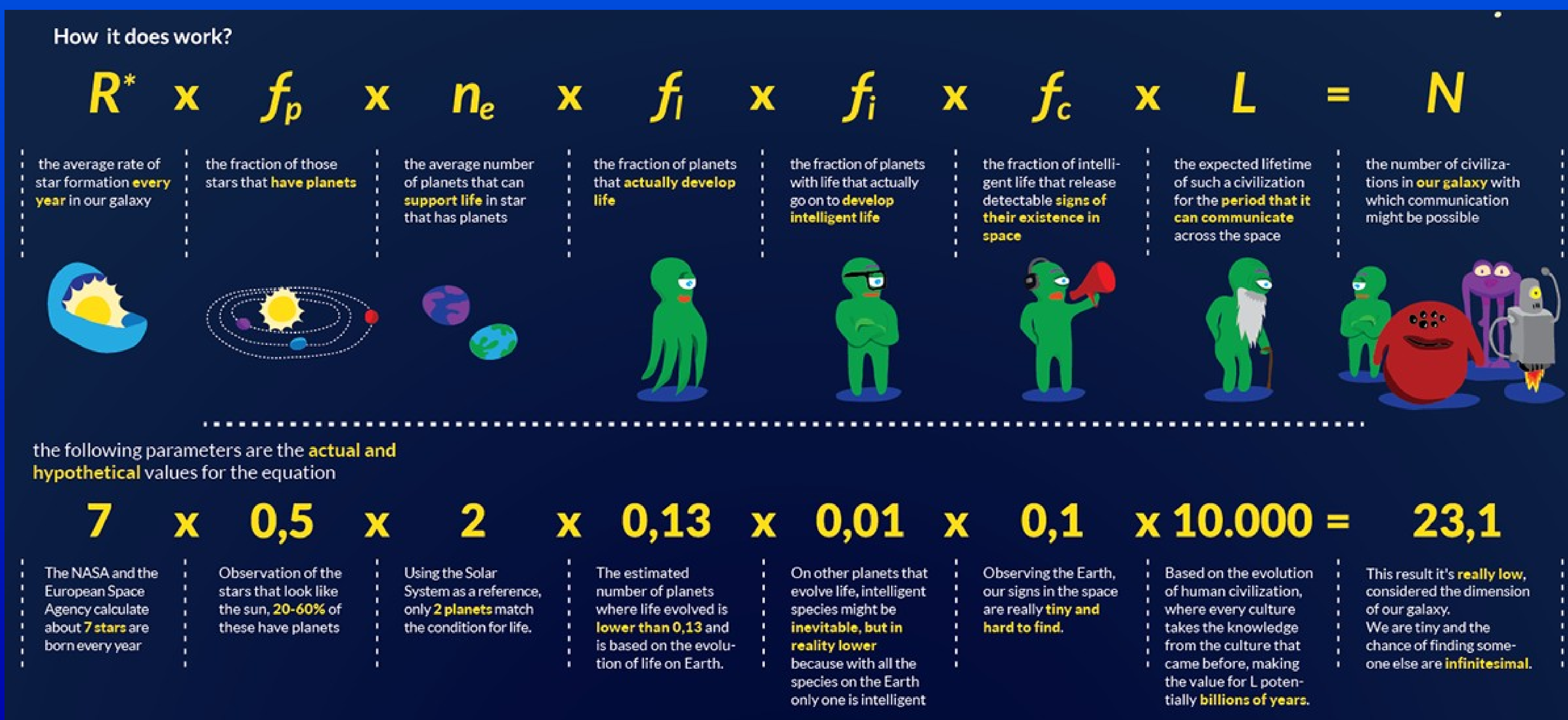

Challenge Problem

How tall is Campbell Hall? What about the Campanile?

Jeopardy!

Metrology	Constants of Nature	Astronomy	Fermi Asks	Cosmology or Cosmotology	Just Physics
100	100	100	100	100	100
200	200	200	200	200	200
300	300	300	300	300	300
400	400	400	400	400	400
500	500	500	500	500	500

For 100 Points:

Kilo-

For 200 Points:

Nano-

For 300 Points:

Tera-

For 400 Points:

Zeta-

For 500 Points:

Yocto-

For 100 Points:

This is the ratio of a circle’s circumference to its diameter.

For 200 Points:

This is the speed of light in m s^{-1}.

For 300 Points:

This is the ratio of proton-to-electron mass.

For 400 Points:

This is the present value of the Hubble constant (which may not be so constant).

For 500 Points:

This is the gravitational constant in SI units.

For 100 Points:

This is the second largest planet in the solar system.

For 200 Points:

This is the angular width, in degrees, of the Moon as viewed from the Earth.

For 300 Points:

With an apparent visual magnitude of -1.46, this is the brightest star in the night sky.

For 400 Points:

This is the mechanism through which stars produce energy in their cores.

For 500 Points:

This is the mass of the Galactic Center black hole in solar masses.

For 100 Points:

This is one day in units of seconds.

For 200 Points:

This is the mass of the human population on Earth in kg.

For 300 Points:

This is the number of grains of rice in a 10 kg bag.

Daily Double:

This is the number of ping pong balls it would take to fill an Olympic size swimming pool.

For 500 Points:

This the weight of a school bus, measured in units of drops of water.

For 100 Points:

Hubble Expansion.

For 200 Points:

Extragalactic Background Light.

For 300 Points:

Extensions.

For 400 Points:

Standard Ruler.

For 500 Points:

Intense Pulsed Light.

For 100 Points:

They are particles that make up neutrons and protons.

For 200 Points:

This describes the effect of gas molecules bumping into each other, and is measured in force per area.

For 300 Points:

This is the law that states entropy can only increase in a closed system.

For 400 Points:

This is the speed needed to escape the Earth's gravity from its surface.

For 500 Points:

This is the law that describes the radiation emitted by an object that absorbs all incident light rays.

ORDER OF MAGNITUDE PHYSICS

Continuing on Basics

RICHARD ANANTUA

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Recap

Some topics we have gone through:

- What is order of magnitude.
- Why it is useful.
- Getting used to thinking in terms of it, and practicing it.

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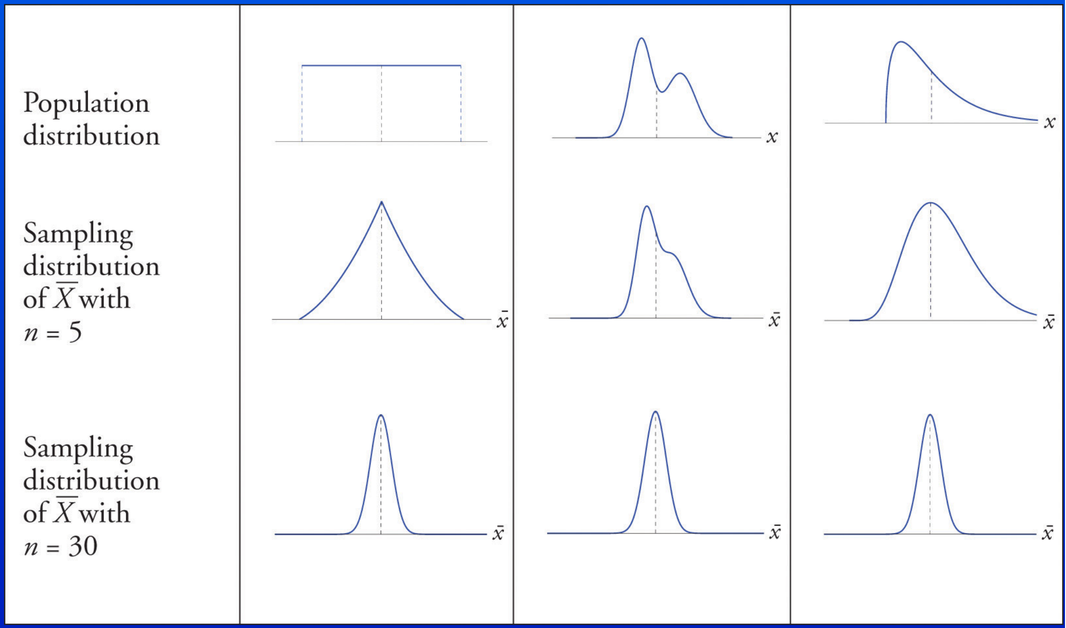
Errors:
The Central Limit Theorem

When independent random variables are added,

their normalized sum tends toward a normal distribution centered on the expected value, with a standard deviation of $\sigma/\sqrt{n}$,

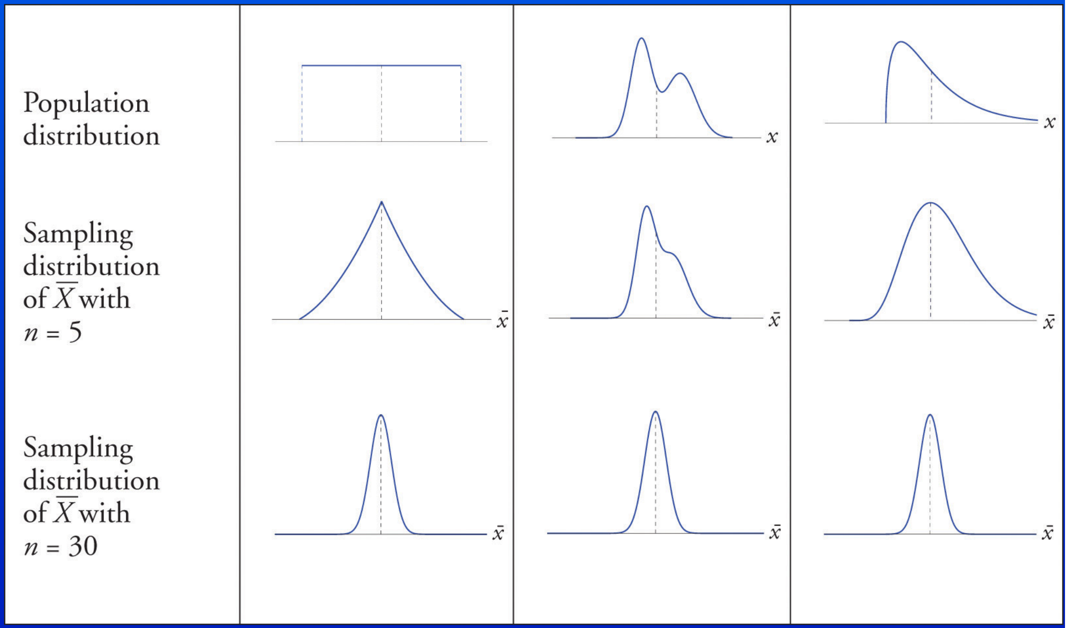


where σ^2 is the variance in the variable.

Errors:
The Central Limit Theorem

If you average *n* independent variables, the error in your average goes down as $\sqrt{n}$.

For multiplication, think of it as addition in the exponent.



When we do order of magnitude calculations, *n* is similar to the number of estimates we used.

Errors: Error Propagation

$$for \quad f(x,y) \quad , \quad \sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2$$

$$if \quad z = x + y$$

$$then \quad \sigma_z^2 = \sigma_x^2 + \sigma_y^2$$

$$if \quad z = x\ y$$

$$then \quad \left(\frac{\sigma_z}{z}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$$

What if $z = e^x$? Can we do order of magnitude estimates with this?

Group Challenge

1. What is the mean density of the Earth in g/cm^3?
2. If there are 100 civilizations in the Milky Way, what is the mean distance between them in light years?
3. How many atoms of iron are there in a sewing needle?
4. How much carbon-dioxide in kg does a human add to the atmosphere each year (by breathing)?
5. How much money is typically transported by an armored car in the US?

Example Solution

How much money is typically transported by an armored car in the US?

Solution 1:

Volume of a ream (500 sheets) of $20 bills:

$$5\,cm \times 7\,cm \times 15\,cm \approx 5\times 10^{-4}\,m^3$$

Volume of the car: $2\,m \times 2\,m \times 2\,m \approx 8\,m^3$

$$\frac{8\,m^3}{5\times 10^{-4}\,m^3} \times 500 \times \$20 \approx \$2\times 10^{8}$$

Example Solution

How much money is typically transported by an armored car in the US?

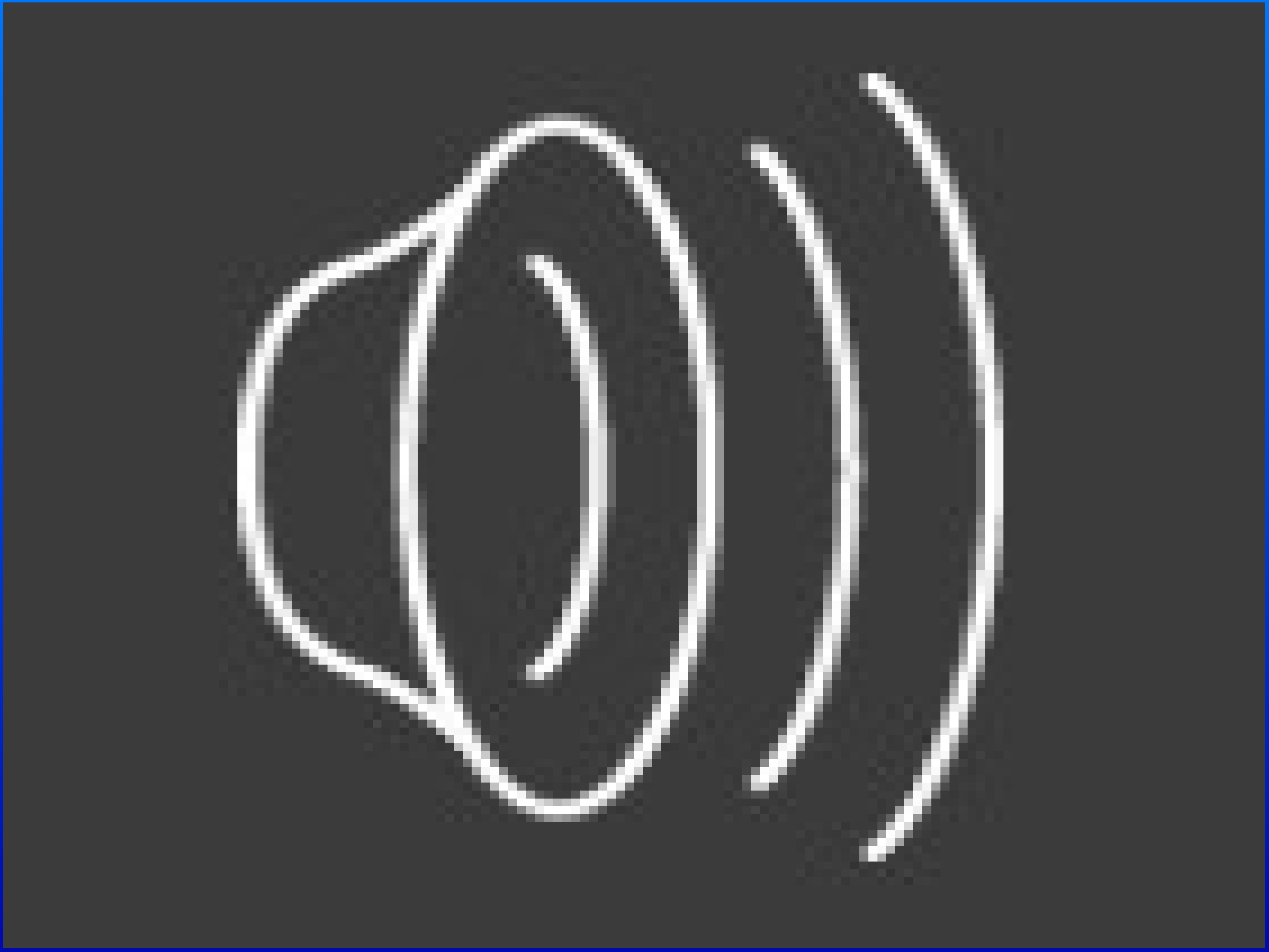
Solution 2:

Mass of a ream (500 sheets) of $20 bills:

$$5\,cm \times 7\,cm \times 15\,cm \times 1\,g/cm^{-3} \approx 500\,g$$

Payload of the car: $about\,1\,tonne = 10^{6}\,g$

$$\frac{10^{6}\,g}{500\,g} \times 500 \times \$\,20 = \$\,2 \times 10^{7}$$



Dimensional Analysis

If you don’t know where to begin, think in terms of units.

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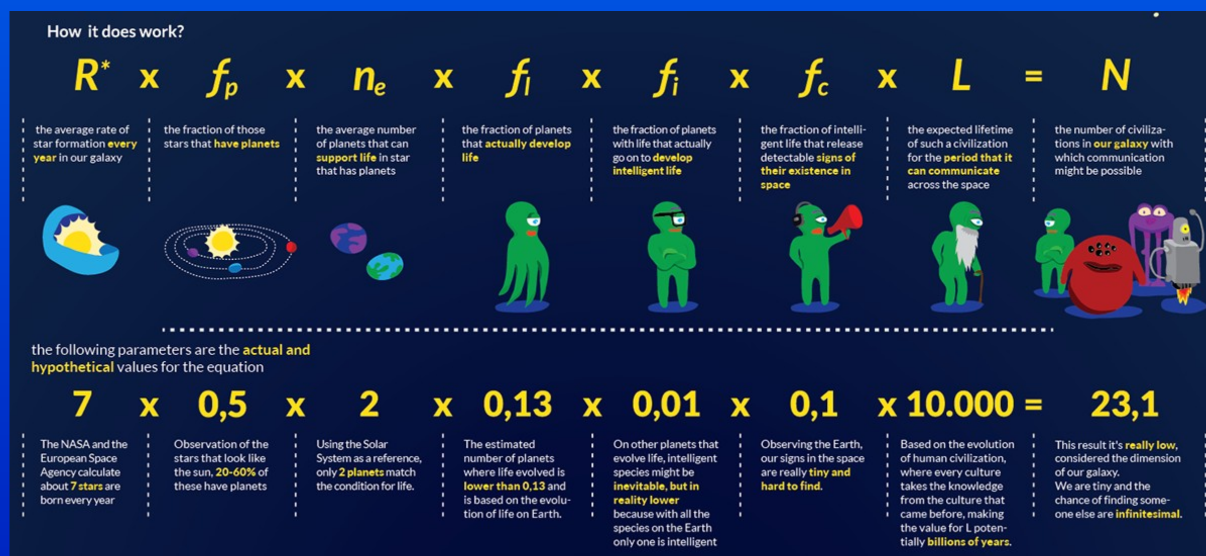

$$\frac{intelligent\ civil.}{years} \times years = intelligent\ civil.$$

$$\frac{stars}{years} \times \frac{planets}{stars} \times \frac{intelligent\ civil.}{planet} \times years = intelligent\ civil.$$

Dimensional Analysis

For every unit, look for a ***characteristic*** value.

Question: What is the speed of a bullet?

$$speed = \frac{distance}{time}$$

$$speed = acceleration \times time$$

$$speed = \sqrt{\frac{energy}{mass}}$$

We can look for some characteristic distance, time acceleration, energy, and/or mass to answer this question.

Dimensional Analysis

For every unit, look for a ***characteristic*** value.

Question: What is the atmospheric pressure at sea level?

Dimensional Analysis

For every unit, look for a ***characteristic*** value.

Question: What is the atmospheric pressure at sea level?

$$pressure = \frac{mass}{length \times time^2}$$

$$pressure = \frac{force}{area} = \frac{mass \times acceleration}{area}$$

$$pressure = \frac{energy}{volume} = \frac{k_B \times temperature}{volume}$$

$$pressure = density \times speed^2$$

The Buckingham Π Theorem

If ***p*** is the number of variables in an equation,

and ***k*** is the number of physical dimensions,

then ***n = p – k*** is the number of dimensionless groups that can be formed from the variables.

$$\Pi_1,\ \Pi_2,\ \Pi_{3,}\ldots\ \Pi_n$$

And the solution is in the form: $C = f\left(\Pi_1,\ \Pi_2,\ \Pi_{3,}\ldots\ \Pi_n\right)$

The Buckingham Π Theorem

Some examples of independent physical dimensions:

- Mass
- Length
- Time
- Temperature
- Charge

The Buckingham Π Theorem

You have 3 variables, Force (F), mass (m) and acceleration (a).
How may they be related?

Variables	Units
F	ML/T^2
m	M
a	L/T^2

There are 3 variables, and 2 (M and L/T^2) physical dimensions.
So there is 3-2=1 dimensionless group.

$\Pi_1 = \dfrac{F}{ma}$ forms the dimensionless group.

$F = ma$ is a possible solution to your problem!

The Buckingham Π Theorem

The simple case of constant acceleration.

Variables	Units
x	L
t	T
v	LT^{-1}
g	LT^{-2}

$$\Pi_1 = \frac{vt}{x} \;,\; \Pi_2 = \frac{g\,t^2}{x} \rightarrow \Pi_1 \Pi_2 = 1 \rightarrow x \approx \sqrt{g\,v\,t^3}\;?$$

The Buckingham Π Theorem

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$$\Pi_1 = \frac{vt}{x} \;,\; \Pi_2 = \frac{g\,t^2}{x} \rightarrow \Pi_1 + \Pi_2 = 1 \rightarrow x \approx g\,t^2 + vt \;?$$

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x	L
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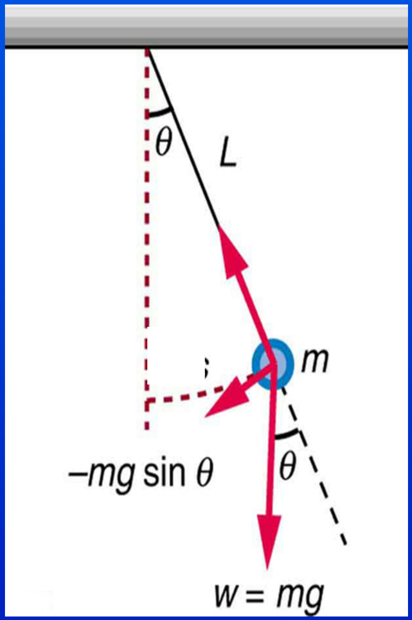
$$\Pi_1 = \frac{vt}{x} \;,\; \Pi_2 = \frac{gt^2}{x} \rightarrow \Pi_1 + \Pi_2 = 1 \rightarrow x \approx gt^2 + vt \;?$$

$$x = \frac{1}{2}gt^2 + v_0 t + x_0$$

The Buckingham Π Theorem

Another simple case of a simple pendulum, which has the solution: $\theta(t) = A\cos(\omega t)$



The Buckingham Π Theorem

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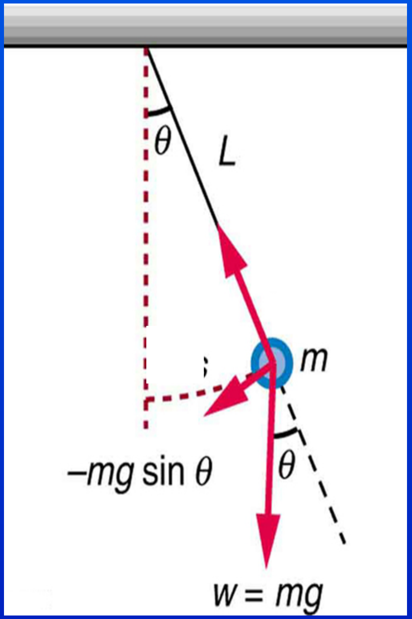
Variables	Units
θ	angle
L	L
m	M
g	LT^{-2}
A	angle
ω	T^{-1}
t	T



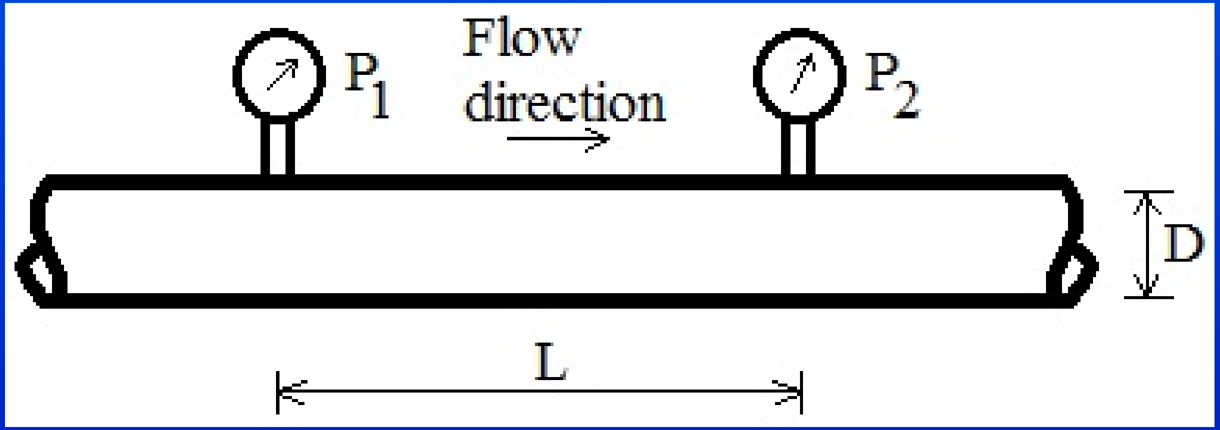
7 variables, 4 dimensions, 3 dimensionless groups.

$$\Pi_1 = \frac{\sqrt{g/L}}{\omega} \;,\; \Pi_2 = \frac{\theta}{A} \;,\; \Pi_3 = \omega t \;\rightarrow\; \omega = \sqrt{g/L}$$

The Buckingham Π Theorem

The pressure drop along a pipe.
How much is the pressure drop $\Delta p = p_1 - p_2$?

The water has viscosity μ [$ML^{-1}T^{-1}$].



The Buckingham Π Theorem

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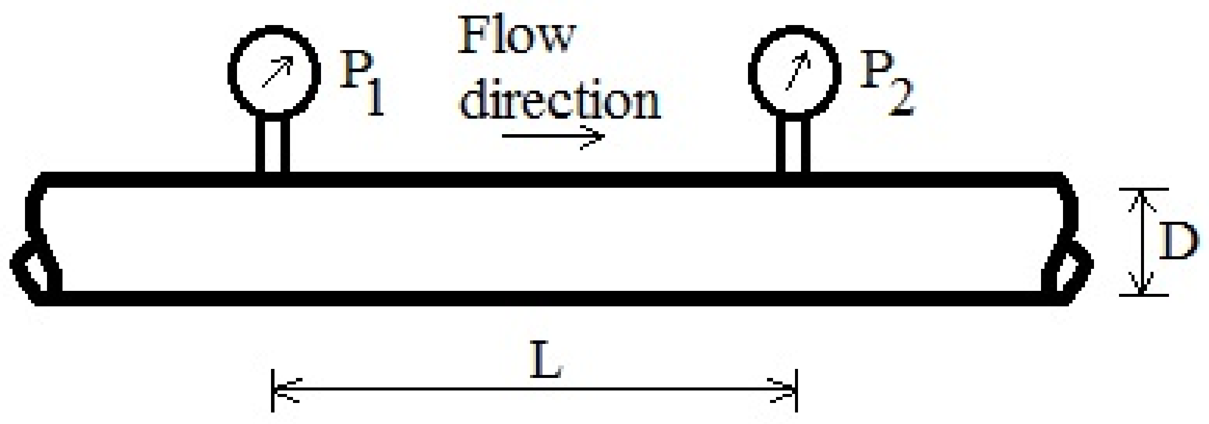
Variables	Units
Δp	$ML^{-1}T^{-2}$
L	L
D	L
v	LT^{-1}
μ	$ML^{-1}T^{-1}$



$$\Pi_1 = \frac{\Delta p\, D}{\mu\, v} \;,\; \Pi_2 = \frac{D}{L} \;\rightarrow\; \Pi_1 \Pi_2 = C$$

$$\Delta p \propto \frac{\mu\, v\, L}{D^2}$$

ORDER OF MAGNITUDE PHYSICS

Continuing on Basics

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Problem set 1 is due right now!

Recap

Some topics we have gone through:

- Errors
  - The Central Limit Theorem
  - Error propagation
- Dimensional Analysis
  - The Buckingham Pi Theorem
- Do order of magnitude estimates before planning a robbery (DON'T PLAN A ROBBERY!)

Group Challenge

1. What fraction of the world's land is covered by human-made structures?
2. How much brighter is the Earth's dayside compared to its nightside?
3. How large of a solar panel do we need to power the entire world?
4. How much water from the ocean is evaporated by the sun every year?
5. How many stars can the naked eye see in the night sky?

A Quick Review: Basics

$F = ma$	Force (Newton's second law).
$p = mv$	Momentum.
$KE = \frac{1}{2}mv^2$	Kinetic energy.
$L = mr^2\omega$	Angular momentum (ω is the angular speed).
$KE_{rot} = \frac{1}{2}I\omega^2$	Rotational kinetic energy (*I* is the rotational inertia).

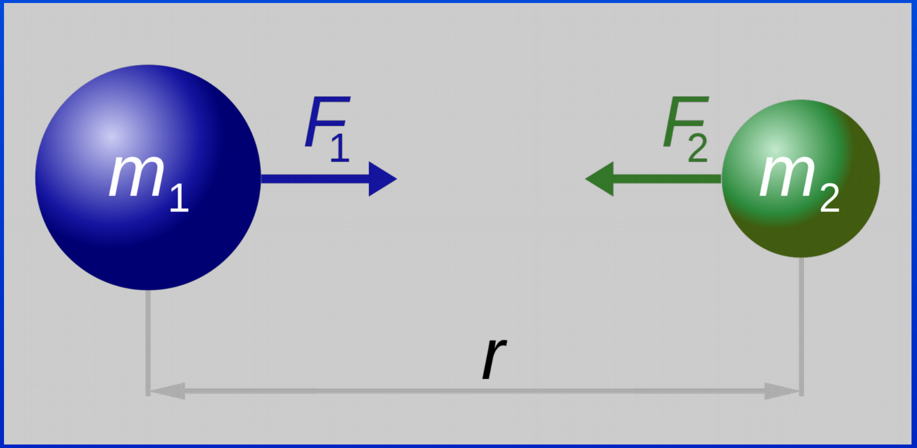
A Quick Review: Gravity

$$G = 6.67 \times 10^{-11} m^3\ kg^{-1}\ s^{-2}$$

Force: $\dfrac{G m_1 m_2}{r^2}$

Acceleration: $\dfrac{G m_1}{r^2}$ or $\dfrac{G m_2}{r^2}$

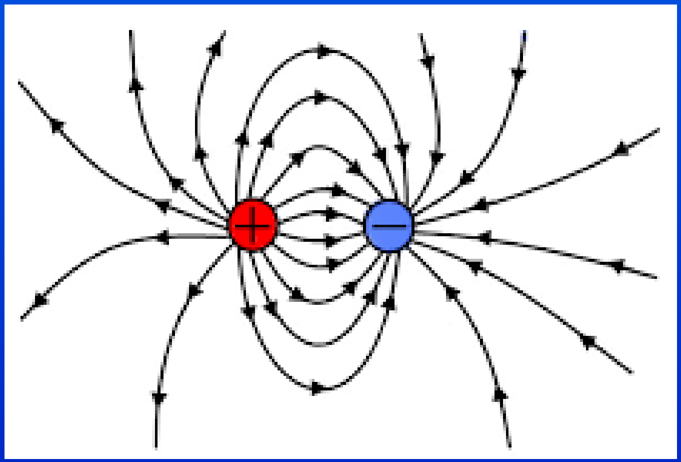
Gravitational Potential: $\dfrac{-G m_1}{r}$ or $\dfrac{-G m_2}{r}$ (Energy per mass)



A Quick Review: Electrostatics (SI units)

$$k_0 = 8.99 \times 10^9 \, N \; m^2 \; C^{-2}$$

A proton charge is $1.6 \times 10^{-19} C$



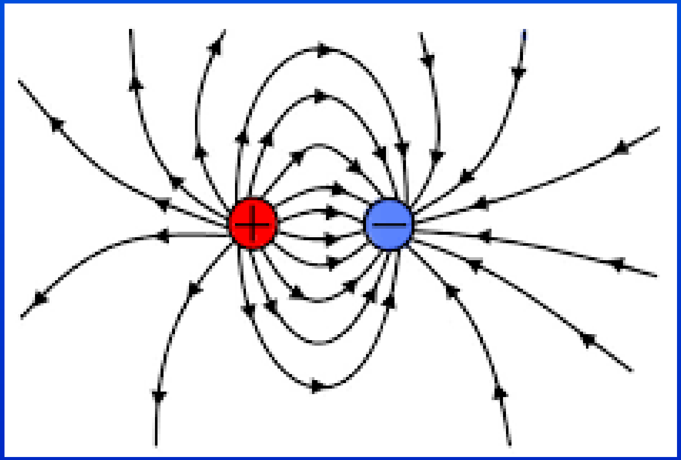
Force: $\dfrac{k_0 q_1 q_2}{r^2}$

Acceleration: $\dfrac{k_0 q_1}{m_2 r^2}$ or $\dfrac{k_0 q_2}{m_1 r^2}$

Electric Potential: $\dfrac{k_0 q_1}{r}$ or $\dfrac{k_0 q_2}{r}$ (Energy per charge)

A Quick Review: Electrostatics (cgs / Gaussian units)

A proton charge is $4.8\times 10^{-10}\, esu$



Force: $\dfrac{q_1 q_2}{r^2}$

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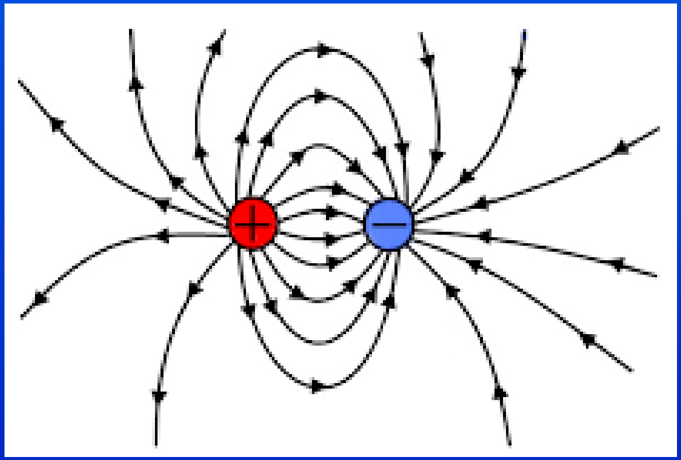

Force: $\dfrac{q_1 q_2}{r^2}$

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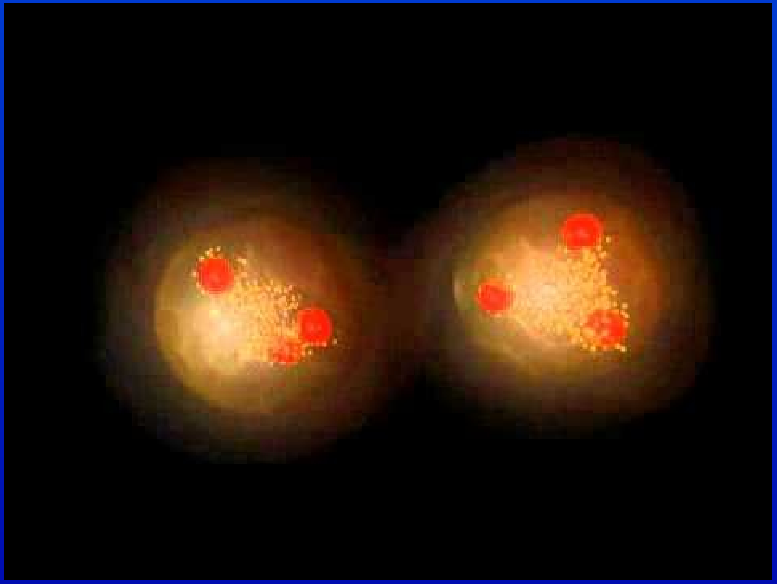
Electric Potential: $\dfrac{q_1}{r}$ or $\dfrac{q_2}{r}$ (Energy per charge)

What are the units of "charge"?

Gravity vs Electric Force

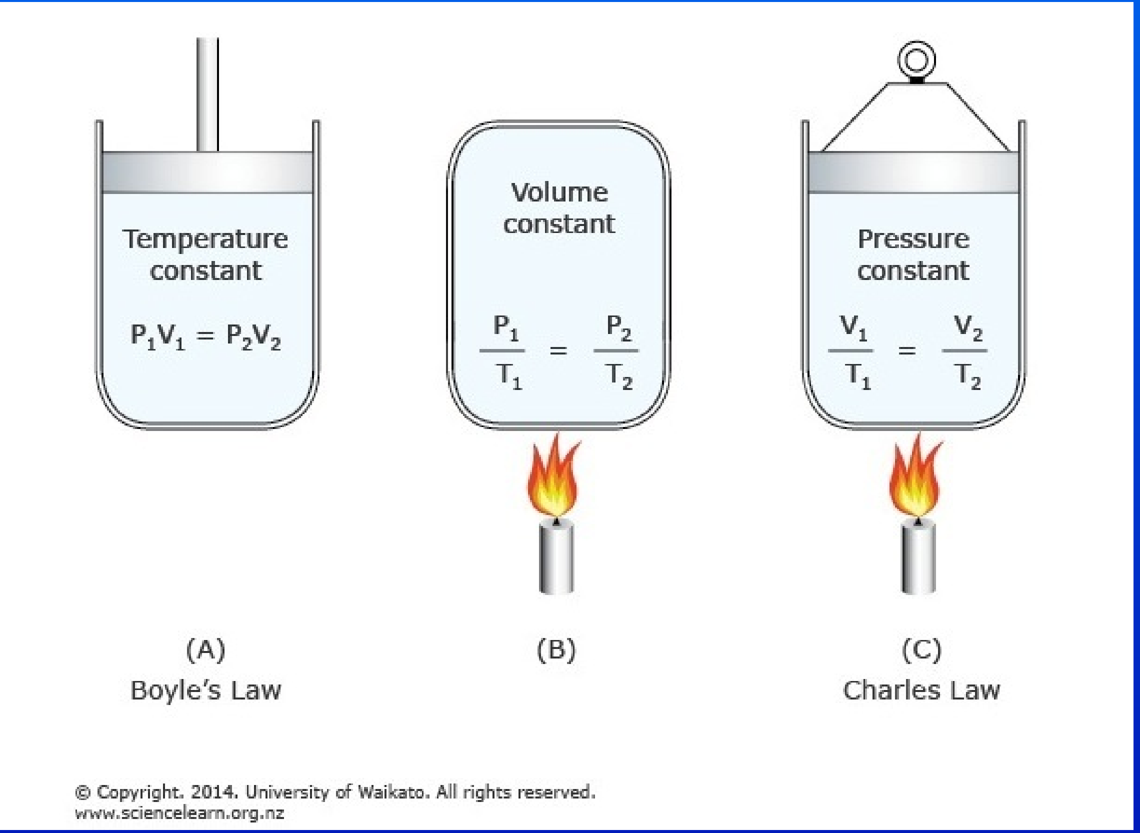
Two protons exert both gravitational and electric forces on each other.

Which force is stronger, and by how much?



A Quick Review: The Ideal Gas Law

$$PV = NRT$$

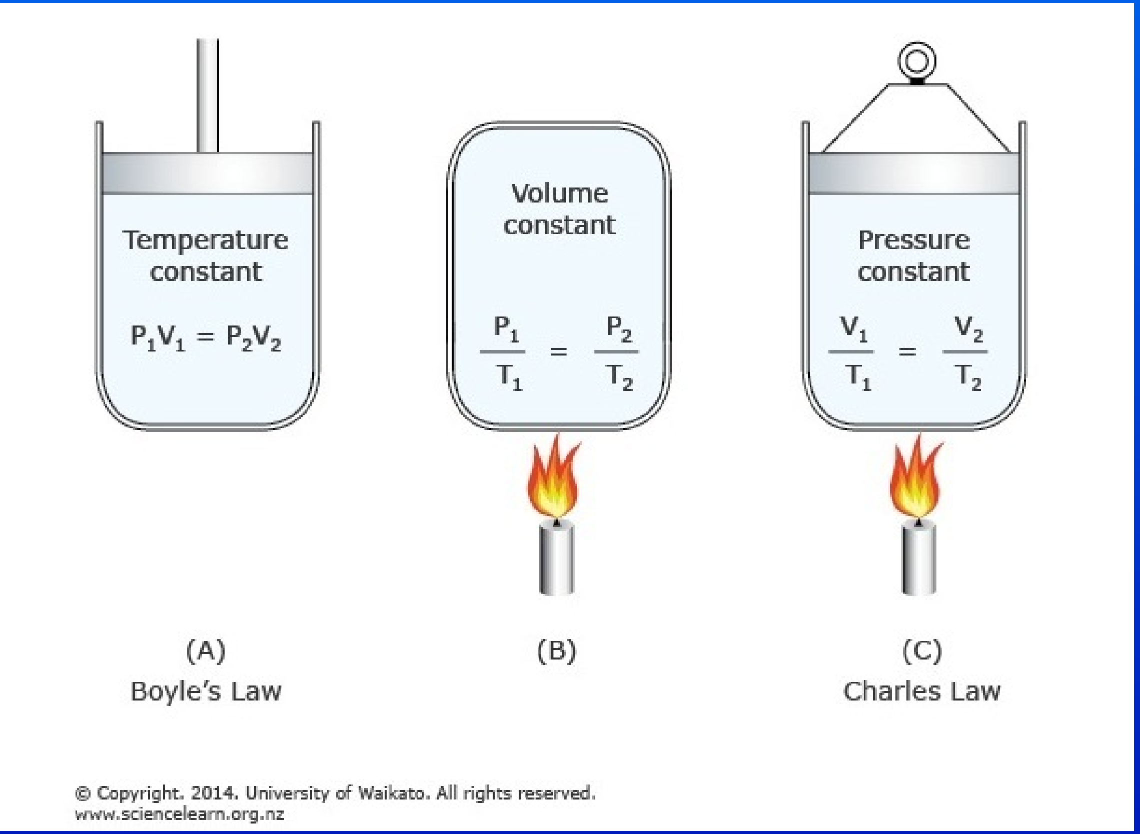


A Quick Review: The Ideal Gas Law

$$PV = NRT$$

$$P = \rho \frac{k_B T}{\mu m_H}$$

$$k_B = 1.38 \times 10^{-23} J\ K^{-1}$$
$$m_H = 1.67 \times 10^{-27} kg$$



A Quick Review: The Ideal Gas Law

$$P = \rho \frac{k_B T}{\mu m_H}$$

$$\frac{P}{n} = k_B T = thermal\,energy\ per\ particle$$

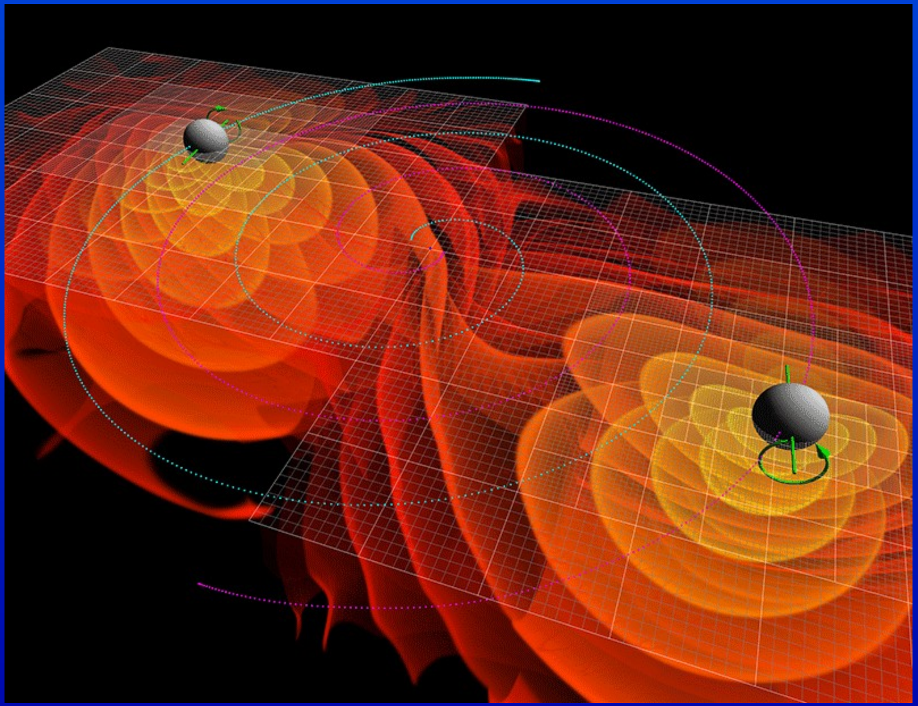
$$P = \rho c_s^2 \ . \ c_s = \sqrt{\frac{k_B T}{\mu m_H}} = isothermal\,sound\,speed$$

A Quick Review:
$E = mc^2$

Special relativity shows energy and mass can be converted into each other.

The first discovery of gravitational wave finds two black holes, one with 35 solar masses and the other 30, colliding. After the collision, 3 solar masses went missing.

How much energy was emitted by the gravitational wave?



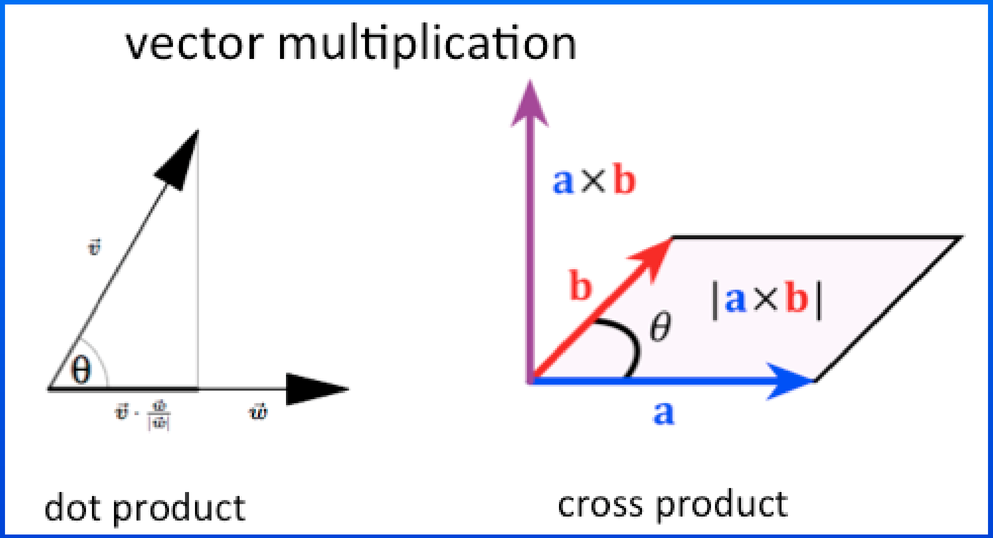
A Quick Review:
Heisenberg's uncertainty principle

$$\Delta x \, \Delta p \geqslant \frac{\hbar}{2}$$

$$\hbar = 1.05 \times 10^{-34} \, J \;\, s^{-1}$$

How uncertain are keys?

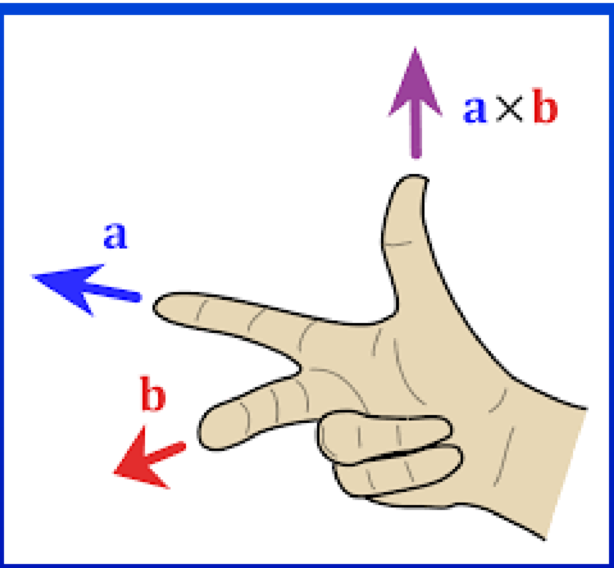
A Quick Review: Vectors



Vectors are multi-dimensional variables,

typically used to store both the magnitude and direction of an object.

$$\vec{a} = \{a_x, a_y, a_z\} \quad , \quad \vec{b} = \{b_x, b_y, b_z\}$$



$$\vec{a} \cdot \vec{b} = a_x\, b_x + a_y\, b_y + a_z\, b_z$$

$$\vec{a} \times \vec{b} = \left(a_y\, b_z - a_z\, b_y\right)\hat{x} + \left(a_z\, b_x - a_x\, b_z\right)\hat{y} + \left(a_x\, b_y - a_y\, b_x\right)\hat{z}$$

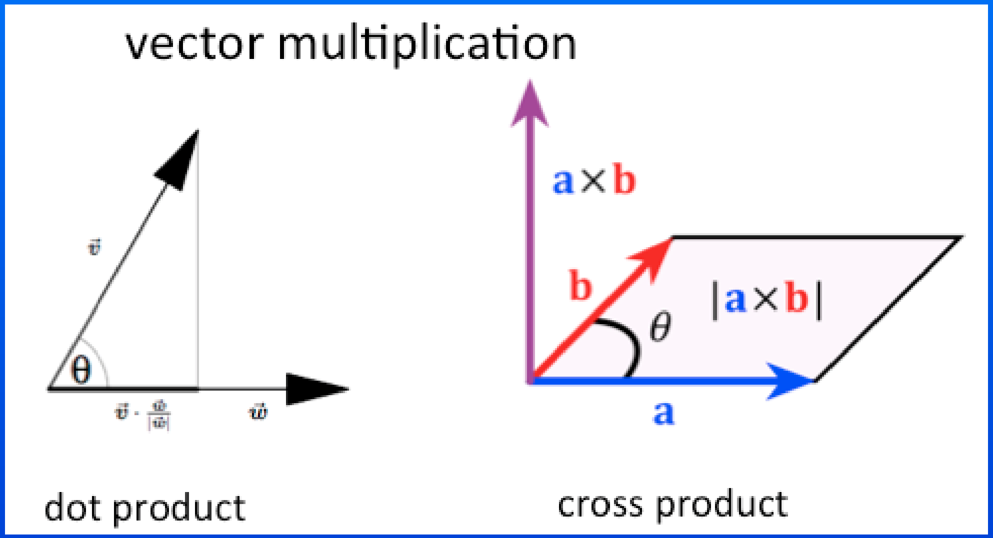
A Quick Review: Vectors

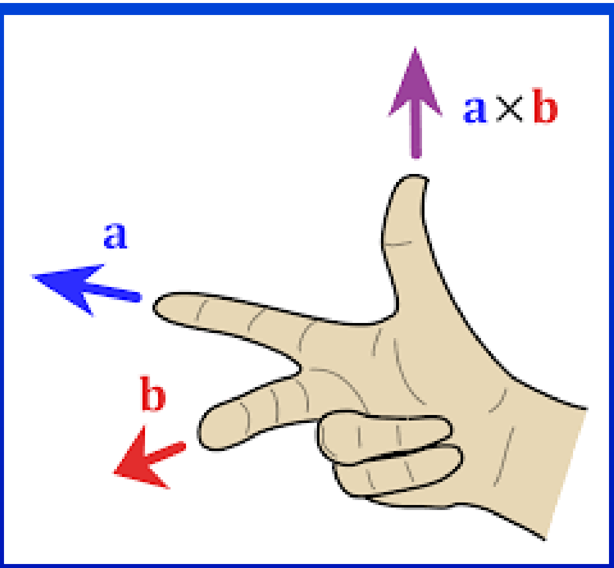
With only two vectors, you can always put them in the same plane to simplify calculations:

$$\vec{a} = \{a_x, a_y\} \quad , \quad \vec{b} = \{b_x, b_y\}$$

$$\vec{a} \cdot \vec{b} = a_x\, b_x + a_y\, b_y$$

$$\vec{a} \times \vec{b} = \left(a_x\, b_y - a_y\, b_x\right) \hat{z}$$





A Quick Review: Taylor Expansion

$$f(x) = f(a) + \frac{df}{dx}\frac{(x-a)}{1!} + \frac{df^2}{d^2x}\frac{(x-a)^2}{2!} + \ldots$$

Two useful relations:

$$\sin\theta = 0 + \theta + 0 - \frac{\theta^3}{6} + \ldots \approx \theta \quad , \quad if \quad |\theta| \ll 1$$

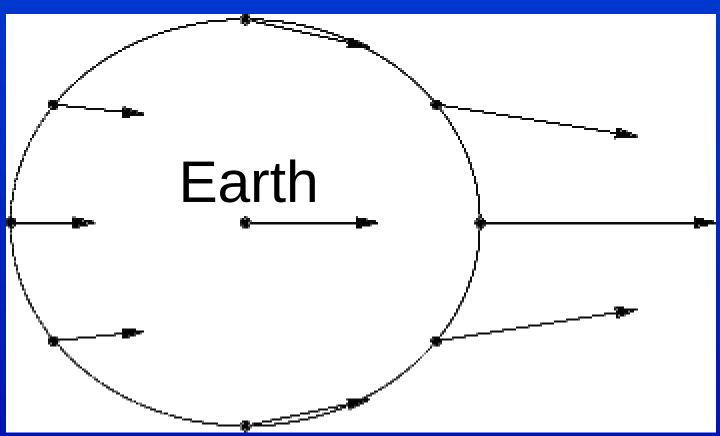
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots \approx 1 + nx \quad ,$$

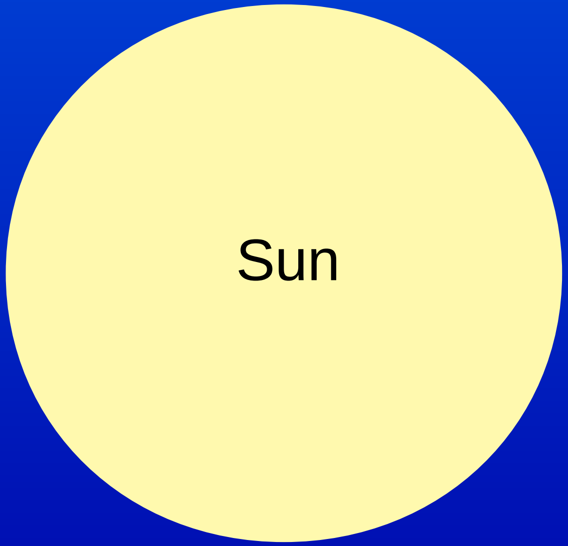
$$if \quad |nx| \ll 1$$

Challenge Problem

One side of the Earth is closer to the Sun than the other side. What is the difference in the gravitational force between the the nearest and farthest points?

What about the Moon?



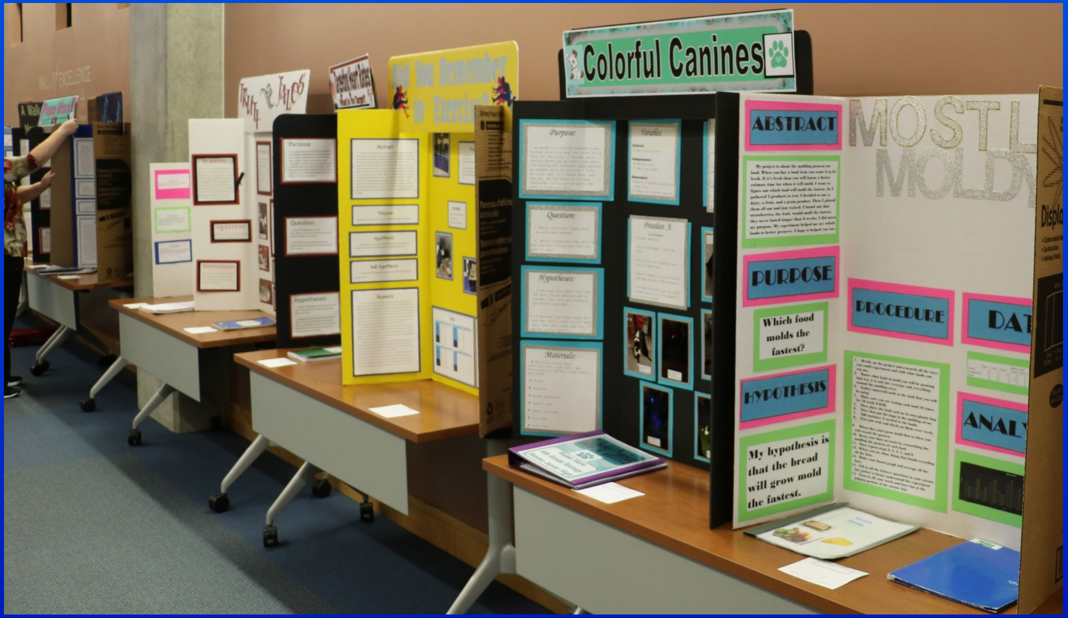


Midterm Project

Order of Magnitude Calculations using Experimental Data

3 to 4 members per group, 2 components, due July 27th

- Poster
  - about 1m x 1m (e.g. A1 paper)
  - No required format. Be creative!



Midterm Project

Order of Magnitude Calculations using Experimental Data

3 to 4 members per group, 2 components, due July 27th

- Presentation
  - 15 mins with slides.
  - All members must participate!

Project Rubric

Order of Magnitude Calculations using Experimental Data

- Poster (10%)
  - Accuracy (4%)
    - Be correct!
  - Thoroughness (3%)
    - Be careful!
  - Flow (3%)
    - Make sense to the reader.
- Presentation (10%)
  - Clarity (4%)
    - Be organized and get to the point.
  - Timing (3%)
    - Use your 15 mins.
  - Teamwork (3%)
    - Everyone does their fair share.
- Overall Depth (10%)
  - Good choice of problem.
  - Well designed experiments.
  - Demonstrates understanding.

In total, 30% of your final grade.

Midterm Project: Brainstorming Session

Step 1

Think about all the physical phenomena you encounter in your daily lives.

Write down questions about them that you may be interested in.

Midterm Project: Brainstorming Session

Step 1

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Step 1

Midterm Project: Brainstorming Session

Step 1

Think about all the physical phenomena you encounter in your daily lives.

Write down questions about them that you may want to answer.

Midterm Project: Brainstorming Session

Step 2

Choose a topic of interest.

Discuss ways to answer those questions.