Universal Density

Thermal Properties.

⇛ Latent heat of Vaporization, Lvap.

Rainfall

$R \simeq 13.6\,eV$

(10% of R) is about the binding energy per molecule.

To evaporate, need to overcome that binding energy.

$$L_{vap} \sim \frac{1\,eV}{1\text{ molecule}} \simeq \frac{1.6\times10^{-12}\,erg}{\text{molecule}}$$

Avogadro number : $N_0 \approx 6\times10^{23}$ /mole.

$1\,cal \approx 4\times10^{7}\,erg$ (Note $1\,Cal = 10^3\,cal$; Cal is used in food label).

$$\boxed{L_{vap}} \approx \frac{1.6\times10^{-12}\,erg}{\text{molecule}} = \frac{1.6\times10^{-12}/(4\times10^{7})\ cal}{1/(6\times10^{23})\ \ mole}.$$

$$= \frac{1.6\times6}{4}\times10^{-12-7+23} \approx \boxed{2.4\times10^{4}\ cal\ mole^{-1}}$$

examples: L_{vap} (10^4 cal mole^{-1})

H_2O	Hg (mercury)	Au (gold)
1.0	1.5	8.1

— Question : how about latent heat of fusion? L_{fus}.

a) long range order destroyed, short range order survives.

also, why does impurity (like salt in ice) makes it easier to melt?

$\left(10^3 \frac{cal}{mole}\right)$	H_2O	Hg	Au
	1.5	0.6	10

May Mention Surface Tension. (sth you can measure by simple experiment)

⇛ Specific Heat ; Heat Capacity ; (negative heat capacity, star-formation)

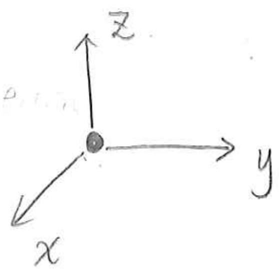
$$\frac{\partial U}{\partial T}$$

U → internal energy

T → temperature.

heat needed to raise the temperature $= \frac{\Delta U}{\Delta T}$

The other way to look at it : if T rises by ΔT with volume unchanged, how much internal energy does increase?



$\boxed{\text{each classically excited degree of freedom contributes } k_B T \text{ to } U}$

therefore $\Delta U \sim 3 k_B \Delta T$. (for a monatomic molecule)

$\boxed{\text{Water \& land}}$

$\therefore C_V \sim \left(\frac{\Delta U}{\Delta T}\right)_{\text{Volume unchanged}} \sim 3k_B$ per molecule.

C: heat capacity; V: volume does not change

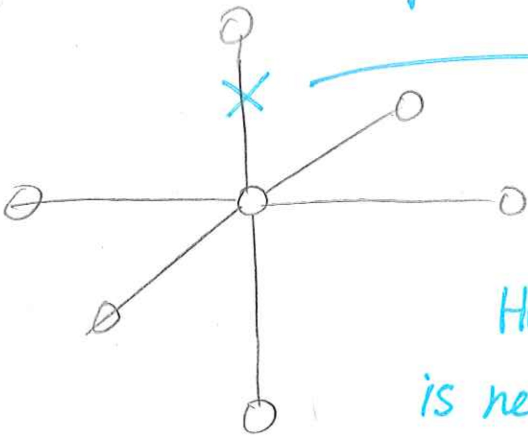
Question : convert to $\frac{erg}{K\cdot cm^3}$

$$\frac{3k_B}{a^3} = \frac{3k_B}{(3\text{Å})^3} \simeq 1.5\times10^7 \frac{erg}{K\,cm^3}$$

$\sim (3k_B N_0)$ per mole $\sim (3\times1.38\times10^{-16}\ erg\ K^{-1})\times(6\times10^{23}\ mole^{-1})$

$\sim 2.5\times10^8\ erg\ K^{-1}\ mole^{-1} \sim 6\ cal\ K^{-1}\ mole^{-1}$.

Motivate Surface Tention.



→ cut bond to make a surface.

→ need energy to do that.

How much energy per unit surface area is needed to cut the surface? The answer is surface tension $\boxed{\gamma = ?\ \frac{\text{erg}}{\text{cm}^2}}$

To cut all the bonds, need about $\left(\frac{1\,\text{eV}}{\text{molecule}}\right)$

Now we only need to cut one bond for each molecule at surface.

$\left(\frac{1}{6} \times \frac{1\,\text{eV}}{\text{molecule}}\right)$.

Surface area of each molecule is about $a^2 \sim (3\,\text{Å})^2$

$$\Rightarrow \gamma = \frac{\frac{1}{6}\,\text{eV}}{(3\,\text{Å})^2} \approx \frac{\frac{1}{6} \times 1.6 \times 10^{-12}\,\text{erg}}{(3 \times 10^{-8}\,\text{cm})^2} \simeq 3 \times 10^2\ \frac{\text{erg}}{\text{cm}^2} \simeq 3 \times 10^2\ \frac{\text{dyn}}{\text{cm}}.$$

correct answer $\boxed{\gamma_{\text{water}} \simeq 50\ \frac{\text{dyn}}{\text{cm}} = 50\ \frac{\text{erg}}{\text{cm}^2}}$

× 6

Question: without heat loss, how fast would the body T rise?

energy input: $P \sim 2000$ Cal / day $\sim 2\times10^6$ cal / day

$\sim 2\times10^6 \times 4\times10^7$ erg / (24 × 3600 seconds)

$\sim 10^9 \ \frac{erg}{s}$

heat capacity per volume $C_V \sim 1.5\times10^7 \ \frac{erg}{K\cdot cm^3}$

Volume: $V = \frac{M}{\rho} \sim \frac{60 \ kg}{1 \ g/cm^3} \sim 6\times10^4 \ cm^3$.

$$\frac{dT}{dt} \sim \frac{\text{energy input power}}{C_V V} \sim \frac{P}{C_V V} \sim \frac{10^9 \ erg \ s^{-1}}{1.5\times10^7 \frac{erg}{K\cdot cm^3} \times 6\times10^4 \ cm^3}$$

$\sim 10^{-3} \ K \ s^{-1} \sim 3.6 \ K/hour \sim 3.6 \ °C / hour$

$\sim 1.8\times(3.6) \ °F / hour \sim 6.5 \ °F / hour$.

40°C (or 104°F) is high fever temperature.

~ 1 hour is long enough to

Human Body must lose heat to survive. What ways does a human body (naked & hairless) lose heat? Question (in class).

1) Radiation

2) Conduction & Advection

3) Evaporative cooling (sweating).

1) Radiation & 3) are relatively easy. Guid to work out in class.

⇒ Radiation: Black body radiation

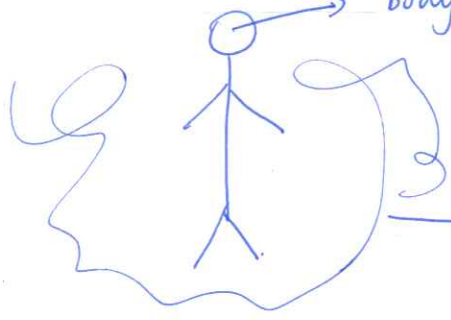
$$\sigma T^4 \approx \left(5.67 \times 10^{-5} \frac{\text{erg}}{\text{s}\cdot\text{cm}^2\cdot\text{K}^4}\right) T^4$$

$\uparrow$

Stefan-Boltzman constant.

Work out this part.

surface area of human body $A \sim 1.5\ m^2 \sim 1.5 \times 10^4\ cm^2$.



→ body temperature $T_{body} \approx 37°C$

→ ambient air $T_{air} \approx 20°C$.

$\boxed{\text{Net}}$ heat loss rate: $L_{radiation} \sim 4 A \sigma T^3 (T_{body} - T_{air})$

$$\sim 10^8 \left(\frac{T_{body} - T_{air}}{1°K}\right) (\text{erg}/\text{s}).$$

3). Evaporative Cooling (Sweating).

a) water loss rate $\dot{M}$

b) Latent heat of evaporization, $L_{vap} \sim 10^4$ cal mole^{-1}

$$\longrightarrow L_{vap} \sim 6\times10^2 \text{ cal g}^{-1} \sim 2.4\times10^{10} \text{ erg/g}.$$

c). $$L_{sweat} \sim \dot{M} L_{vap} \sim \left(\frac{\dot{M}}{\text{kg h}^{-1}}\right) \frac{10^3 \text{ g}}{3600 \text{ seconds}} \times 2.4\times10^{10} \frac{\text{erg}}{\text{g}}$$

$$\sim 6.7\times10^9 \left(\frac{\dot{M}}{\text{kg h}^{-1}}\right) \text{ erg/s}$$

guess $\dot{M} \sim$ kg h^{-1} from experience of exercising, hiking.

⇒ Thermal Diffusivity.

phonon mean free path

sound speed.

1) by lattice phonons: $\boxed{K_p \sim \frac{\lambda_p C_s}{3} \sim \frac{(3a)\times(3\,km/s)}{3}}$

p denotes "phonon"

velocity along random direction

$$\sim \frac{(3\times 3\,\mathring{A})\times(3\,km/s)}{3} \simeq (3\times10^{-8}\,cm)\times(3\times10^{5}\,cm/s) \approx 10^{-2}\,\frac{cm^2}{s}.$$

a) mean free path λ_p is several lattice spacings.

b) phonon-phonon scattering yields $\lambda_p \propto T^{-1}$

? Question: $\boxed{\text{Cooking time}} \sim \text{length}^2/K_p \sim 10^2\left(\frac{\text{length}}{cm}\right)^2$ seconds.

2). Metal is a much better thermal conductor, because heat gets diffused by fast e^-. (at surface of Fermi sea)

$$K_e \sim \frac{\lambda_e v_F}{3} \sim \frac{(100\,a)\times(10^3\,km/s)}{3} \sim 10^2\,cm^2\,s^{-1}.$$

electron

extra material.

? Question: thermal inertia of ⊕ (earth) why 2pm hottest land?

2) by air molecules's thermal velocity: (move this to later).

$$K_{air} \sim \frac{\lambda_{air} v_{air}}{3}$$

$$v_{air} \sim \sqrt{\frac{3k_BT}{m_{air}}} \sim \sqrt{\frac{3k_BT_{ROOM}}{30m_H}} \sim \sqrt{\frac{3k_B\times 300°K}{30\,m_H}} \sim 0.3\,km/s.$$

number of air molecules per volume

mean free path: $\lambda_{air} \sim (n_{air}\times\sigma)^{-1}$

cross section for molecule collision

$$d \sim \left(\frac{K_{th}}{\omega_{day}}\right)^{1/2} \sim \left[\frac{0.01\,\frac{cm^2}{s}}{(2\pi/10^5\,s)}\right]^{1/2} \sim \left(\frac{10^3}{2\pi}\right)^{1/2} \simeq 12\,cm.$$

$$\frac{C_v\, d\, \cancel{A}\, \Delta T}{F\, \cancel{A}} \simeq \frac{2\times10^7 \frac{erg}{K\cdot cm^3} \times 12\,cm \times 5\,°K}{5.67\times10^{-5} \underbrace{}_{\frac{erg}{cm^2\cdot s\cdot K^4}} \times (300\,K)^4} \approx \frac{1.2\times10^9}{4\times10^5} \approx 3000\,s.$$

$$F \approx \sigma T^4 \approx 4.6\times10^5\ \frac{erg}{cm^2\cdot s}$$

$\approx$ 1 hour.

roughly right.

hottest ground around 2pm

Briefly mention: moon's surface is dusty.

(porous material).

$$\Gamma \equiv \rho C_p (K)^{1/2}.$$

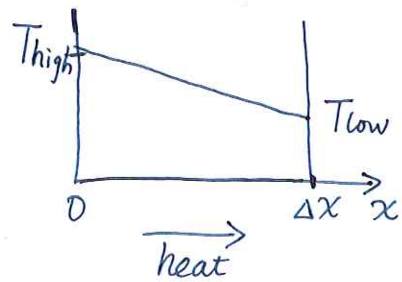
$$\lambda_{air} \sim (n_{air}\,\sigma)^{-1} \sim \left[\left(\frac{\rho_{air}}{30\,m_H}\right)\sigma\right]^{-1} \sim \left[\left(\frac{10^{-3}\,g/cm^3}{30\,m_H}\right)\pi a^2\right]^{-1}$$

$$\sim \left[2\times 10^{19}\,cm^{-3} \times \pi (3\,\text{Å})^2\right]^{-1} \sim 1.7\times 10^{-5}\,cm.$$

$$\Rightarrow K_{air} \sim \frac{\lambda_{air}\, v_{air}}{3} \sim \frac{(1.7\times 10^{-5}\,cm)\times(0.3\,km/s)}{3} \sim 0.17\ cm^2/s$$

$$\boxed{K_{air} \sim 0.2\ cm^2\,s^{-1}}$$

$\Rightarrow$ Thermal conductivity :



heat transfers from Thigh to Tlow ; Given a temperature gradient $\frac{dT}{dx} = \left(\frac{T_{high} - T_{low}}{\Delta x}\right)$, what is the heat flux, which is the energy transfered by conduction per unit time per unit area.

$$F = k_{cond} \frac{dT}{dx}$$

F: heat flux $\frac{erg}{cm^2 \cdot s}$

k_{cond}: conductivity — its unit should be ? Question? $\frac{erg}{cm \cdot s \cdot K}$

$\frac{dT}{dx}$: temperature gradient $\frac{°K}{cm}$

Question :

can you compose the correct expression for k_{cond}, using physical quantities we just talk about ?

Relevant Variables	Unit
~~F (?)~~	
~~T (?)~~	
K	$cm^2\ s^{-1} = [L]^2[T]^{-1}$
C_V (heat capacity per volume)	$\frac{erg}{cm^3 \cdot {}^\circ K} = [M][L]^{-1}[T]^{-2}[Tem]^{-1}$
k_{cond}	$\frac{erg}{cm \cdot s \cdot K} = [M][L][T]^{-3}[Tem]^{-1}$

$K C_V$ has the same dimension as k_{cond}.

Indeed, $\boxed{k_{cond} = K C_V}$

1) Lattice Phonon : $C_{V,bulk}$ $1.5 \times 10^7 \frac{erg}{K \cdot cm^3}$

$K_p \sim 10^{-2} \frac{cm^2}{s}$

$\Rightarrow k_{cond,p} \sim 1.5 \times 10^5 \frac{erg}{cm \cdot s \cdot K}$

2) air : $C_{V,air} \sim C_{V,bulk} \left(\frac{\rho_{air}}{\rho_{bulk}}\right) \sim 1.5 \times 10^4 \frac{erg}{K \cdot cm^3}$

$K_{air} \sim 0.2\ cm^2\ s^{-1}$

$\Rightarrow k_{cond,air} \sim 3 \times 10^3 \frac{erg}{K \cdot cm^3}$

right number.

$\boxed{2 \times 10^3 \frac{erg}{K \cdot cm^3}}$

Guid for Homework problem ~~cor~~ about cooling rate of human body due to conduction & advection.

I) Do we need to worry about the conduction inside human body?

No. Because $k_{cond,p} >> k_{cond,air}$, conduction is limited by conduction of the air. The human body can be well approximated by a uniform-T body.

II) Which way makes you feel cooler?

Stand, walk, or Run?

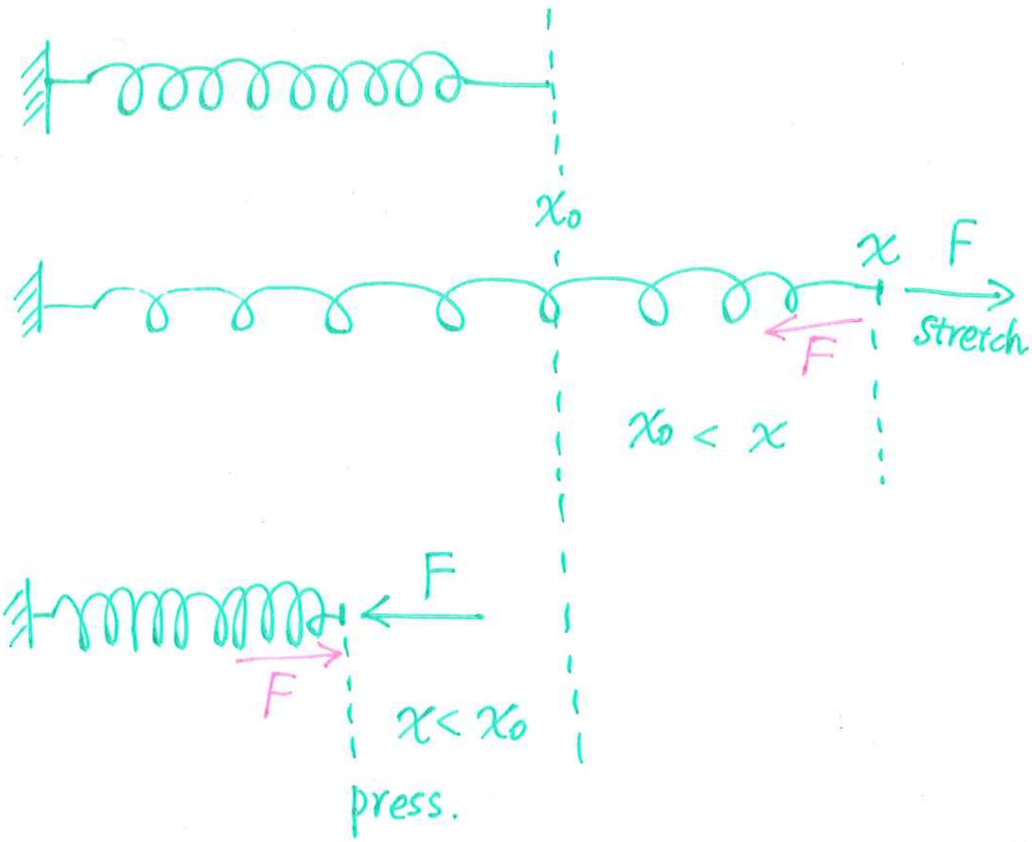
Why does speed matter? Replacement of fresh cool air around you. (Advection)

How does speed come in?

Mechanical Properties

Context: Sitting on a chair ; Press table;

Spring

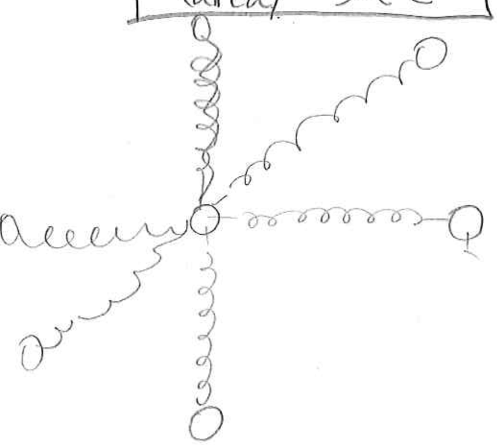


1) Intuition : $F \propto \left(\frac{x-x_0}{x_0}\right)$ fractional change of length

generalized concept : strain ϵ

Sitting on a chair

$$\boxed{\left(\frac{F}{\text{area}}\right) = \mu \epsilon}$$



What's the coefficient μ ? $\boxed{\text{elastic modulus}}$. $\left[\frac{\text{erg}}{\text{cm}^3}\right.$ or $\left.\frac{\text{dyn}}{\text{cm}^2}\right]$

$$\boxed{\text{Reminder : } L_{vap} \simeq \frac{1\,\text{eV}}{a^3}}$$

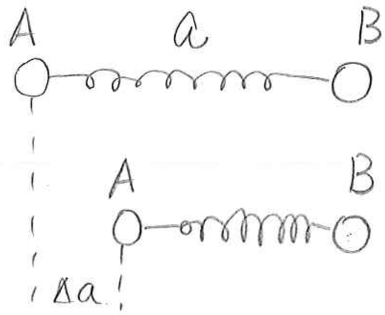
Show them this figure & remind them the way we estimate L_{vap}. How would you attach this questi

$$\mu \simeq \frac{1\,\text{eV}}{a^3} \simeq \frac{1.6\times 10^{-12}\,\text{erg}}{(3\times 10^{-8}\,\text{cm})^3} \simeq 6\times 10^{10}\ \frac{\text{erg}}{\text{cm}^3}\ \left(\frac{\text{dyn}}{\text{cm}^2}\right)$$

⇒ Sound Speed : elastic wave speed. (sound speed)

~~exercise~~ ?

Seismology.



$\left(a^2 \times \frac{\Delta a}{a} \times \mu\right)$ Force.

$m = \rho a^3.$ Mass

$$\text{Acceleration} = \frac{\text{Force}}{\text{mass}} \approx \frac{\mu}{\rho} \frac{1}{a} \left(\frac{\Delta a}{a}\right)$$

Start with zero speed, how long does it take for B to move the same distance (Δa) as A ?

$$\frac{t^2}{2} \times \text{Acceleration} \simeq \Delta a$$

$$t^2 \sim a^2 \left(\frac{\rho}{\mu}\right)$$

The speed for the wave to go from A to B is

$$C_s \sim \frac{a}{t} \sim \left(\frac{\mu}{\rho}\right)^{1/2} \sim \left(\frac{10^{11}}{1}\right)^{1/2} \sim (3\ \text{km/s}).$$

Compared to sound speed of air $C_{s,\text{air}} \approx 300$ m/s $\approx$ 10 time smaller than that of solid.

Example : sound travel time through the earth ?

$(2R \sim 10^4$ km$) / (3$ km/s$) \sim 3000$ s. $\sim$ 1 hour.

examples. μ (10^{11} dyn cm^{-2})

Al	Steel	wood	bone	diamond	ice (water)
7	20	1	2.	100	0.4. ~0.8

Wait (first introduce yield stress.). $Y \sim (10^{-3} \sim 10^{-2})\mu$

Question: Max size for irregular shaped asteroid?
elastic modulus against gravity.

Max height for mountain on earth?

mountain: $\rho g h < \epsilon_Y \mu$

$$h < \frac{\epsilon_Y \mu}{\rho g} \sim \frac{10^{-3\sim-2} \times 10^{11}}{1 \times 10^3} \simeq (1 \sim 10)\ \mathrm{km}.$$

tallest mountain on earth. (Mount Everest)

$h \simeq 8848\ \mathrm{m} \approx 8.848\ \mathrm{km}.$

Tallest mountain on Mars? $g_{Mars} \approx 3.7\ \mathrm{m/s^2}$

$\approx 0.4\, g_{earth}.$

Mars: Olympus Mons $h \approx 25$ km.

Asteroid: irregular-shaped ($h \sim R$)

$\rho g R \sim \epsilon_Y \mu$ $\quad \because g = \frac{GM}{R^2} = \frac{4\pi G \rho}{3} R$

$\therefore \frac{4\pi}{3} G \rho^2 R^2 \sim \epsilon_Y \mu.$

$\therefore R \sim 6 \times 10^7$ cm ~ 600 km

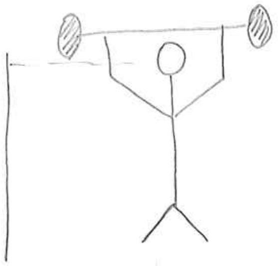
about right

$\sim 10^2$ km

Example: Asteroseismology → Radius, He core, → Age

seismic wave in earth.

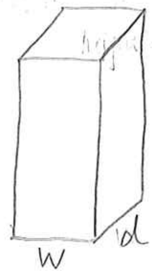
Weight Lifting $\propto R^2 \propto M_{human}^{2/3}$.



mass of the weight: m_{weight}.

compression of human body: $\left(\frac{\Delta h}{h}\right)$

human



Stress $\Rightarrow$ $\sigma \sim \mu \left(\frac{\Delta h}{h}\right)$

must balance.

Force $\Rightarrow$ $F \sim \sigma (wd) \sim \mu \left(\frac{\Delta h}{h}\right) wd \;=\!=\; m_{weight}\, g$.

μ & maximum $\frac{\Delta h}{h}$ are universal.

therefore, $\boxed{m_{weight} \propto wd \propto M_{human}^{2/3}}$

guess

$$m_{weight} \sim \frac{\mu w d}{g}\left(\frac{\Delta h}{h}\right) \sim \frac{\left(3\times10^{10}\,\frac{dyn}{cm^2}\right)\times(0.5\,m\times0.5\,m)}{10\ m/s^2}\times10^{-6}\left(\frac{\Delta h/h}{10^{-6}}\right)$$

$$\sim \frac{(3\times10^{10})\times(0.25\times10^{4})}{10^3}\times10^{-6}\times\left(\frac{\Delta h/h}{10^{-6}}\right)\ \text{grams}$$

$$\sim 0.75\times10^{5}\left(\frac{\Delta h/h}{10^{-6}}\right)\ \text{grams} \sim 100\left(\frac{\Delta h/h}{10^{-6}}\right)\ kg$$

video ; ant.