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Constraining the chiral magnetic effect's ability to generate magnetic fields in proto-neutron stars

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MOTIVATION + CONTEXT

- A neutron star forms as the remnant of a massive (10 – 25 $M_\odot$) star's core after a supernova explosion
- Magnetars are neutron stars with incredibly strong magnetic fields (10^{14}–10^{16} Gauss; the strongest magnetic fields in the universe), but the mechanisms by which these fields are produced are currently poorly understood

Artist's impression of a magnetar (Quanta Magazine)

- A possible explanation for these fields is the operation of a dynamo during the proto-neutron star (PNS) phase; candidates include convection, differential rotation, and the chiral magnetic effect (CME)
- During neutronization (p + e -> n + ν), an asymmetry in number density of left- vs. right-handed electrons arises, generating a large chemical potential μ; the CME converts μ to magnetic energy
- This study uses analytical approaches and chiral magnetohydrodynamic (MHD) simulations [run in Dedalus, a Python numerical PDE solver] to understand the characteristic timescales and maximum achievable fields from this process
- Specifically addressing a realistically slow (wrt. dynamo timescale) buildup of chemical potential μ; previous literature has overlooked this point (assumes μ is instantly generated)

CHIRAL MHD IN NUMERICAL SIMULATIONS

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times [\underbrace{\boldsymbol{U} \times \boldsymbol{B}}_{\simeq E} - \underbrace{\eta}_{\text{microscopic magnetic diffusivity}}(\nabla \times \boldsymbol{B} - \underbrace{\mu \boldsymbol{B}}_{\text{linear term}})] \quad (1) \text{ induction equation}$$

$$\rho \frac{D\boldsymbol{U}}{Dt} = \underbrace{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}_{\text{Lorentz force}} - \nabla p + \nabla \cdot \underbrace{(2\nu\rho \mathbf{S})}_{\text{viscosity}} + \underbrace{\rho \boldsymbol{f}}_{\text{external forces}} \quad (2) \text{ Navier-Stokes equation}$$

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \boldsymbol{U} \quad (3) \text{ continuity equation}$$

$$\frac{D\mu}{Dt} = D_5 \Delta\mu + \underbrace{\lambda}_{\text{inverse susceptibility}} \eta [\boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) - \mu \boldsymbol{B}^2] - \underbrace{\Gamma_f \mu}_{\text{chiral flipping}} + \underbrace{\dot{\mu}_0}_{\text{forcing from neutronization}} \quad (4) \text{ CME}$$

$$\frac{\lambda}{2} \underbrace{\langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle}_{\text{helicity}} + \underbrace{\langle \mu \rangle}_{\text{chiral chemical potential}} = \mu_0 = \text{const.} \quad (5) \text{ chirality conservation}$$

- Eqs. (1)-(3) above are the traditional MHD equations, with Eq. (4) and the highlighted term in Eq. (1) being the additions that account for the CME, quantified by the chiral chemical potential μ
- Eq. (5) arises from the other four and demonstrates conservation of total chirality
- When the highlighted term in Eq. 1 dominates, B-field grows exponentially (linear PDE); maximum growth rate in this phase is $\gamma_{max} = (\mu^2\eta)/4$ (characteristic timescale)
- Other characteristic scales are $B_{sat} = \mu/\sqrt{\lambda}$ and the nonlinearity parameter $\chi = \lambda\eta^2$
- Neutronization rate is quantified by $\dot{\mu}_0$ term in Eq. 4 – assumed to be constant linear forcing

PRELIMINARY RESULTS

When the velocity field is small compared to the B-field (linear regime), we derive and numerically verify a **1/3 power scaling** of **B-field with neutronization rate** $\dot{\mu}_0$. Using values of μ set by chemical potential and λ from HEP [2], we estimate an upper bound of **10^{12} Gauss** for the magnetic field.

DERIVATION OF SCALING

Saturation is achieved when Dμ/Dt = 0. From Eq. 4, we have:

$$\lambda\eta[\boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) - \mu \boldsymbol{B}^2] + \dot{\mu}_0 = 0$$

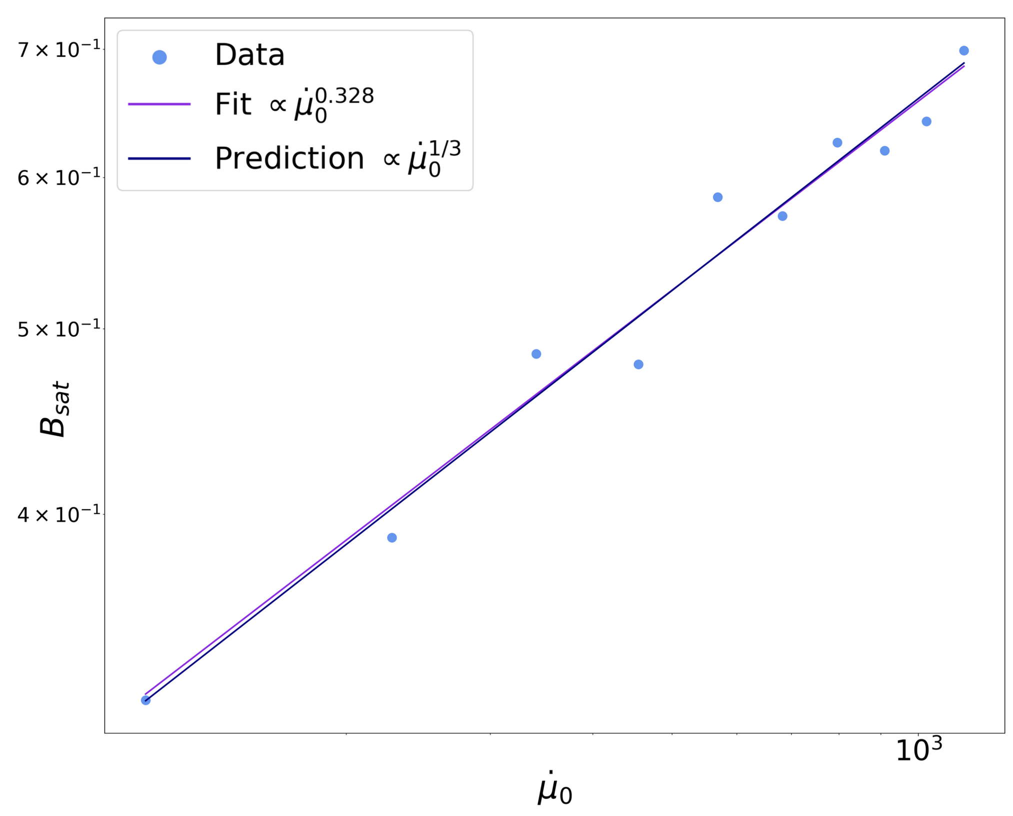
Since $\boldsymbol{B} \sim e^{\frac{\mu \vec{x}}{2}}$, $\boldsymbol{B} \cdot (\nabla \times \boldsymbol{B}) \sim \frac{\mu \boldsymbol{B}^2}{2}$, we have:

$$\lambda\eta \frac{\mu \boldsymbol{B}^2}{2} = \dot{\mu}_0 \implies \boldsymbol{B}^2 = \frac{2\dot{\mu}_0}{\lambda\eta\mu}$$

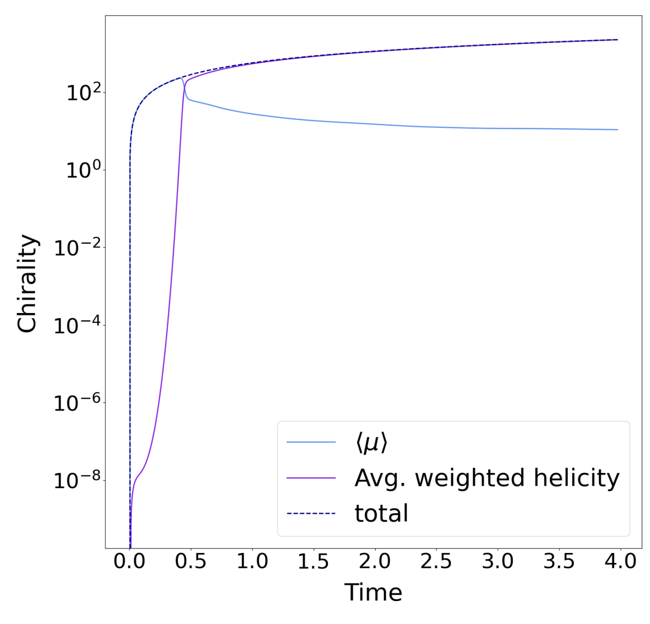
From Eq. 5 (chirality conservation) and using $\boldsymbol{A} \sim \frac{\boldsymbol{B}}{\mu}$:

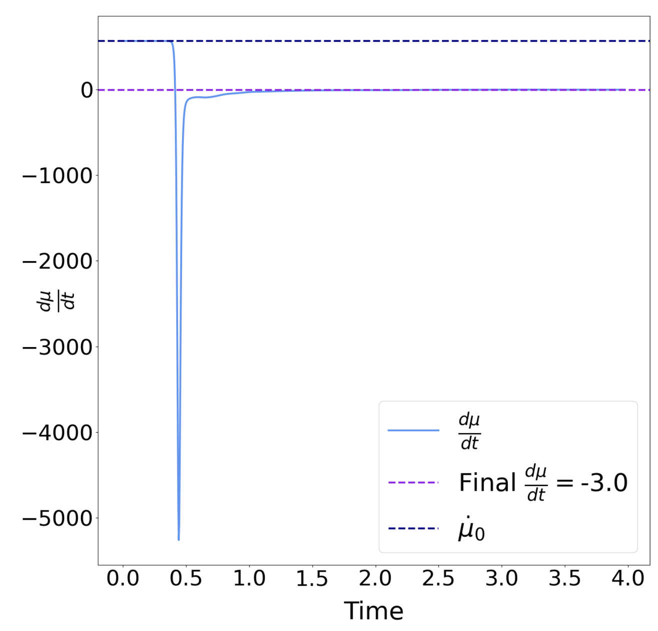
$$\frac{\lambda}{2}\left(\frac{\boldsymbol{B}^2}{\mu} + \frac{2\mu}{\lambda}\right) = 0 \implies \frac{2\dot{\mu}_0}{\lambda\eta\mu^2} = \frac{2\mu}{\lambda}$$

$$\mu = \left(\frac{\dot{\mu}_0}{\eta}\right)^{\frac{1}{3}}, \boldsymbol{B} = \frac{1}{\sqrt{\lambda}}\left(\frac{\dot{\mu}_0}{\eta}\right)^{\frac{1}{3}}$$

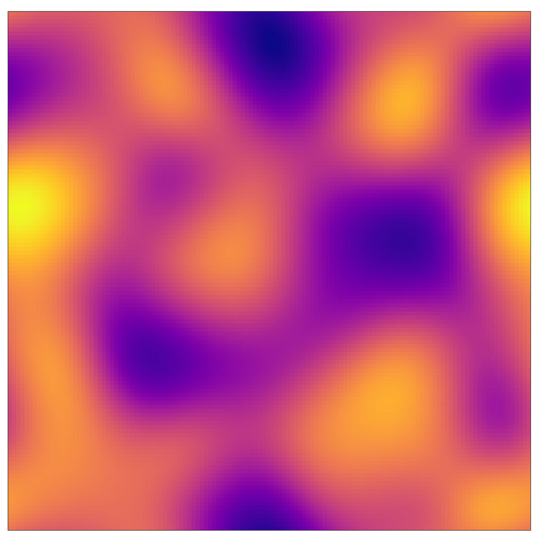


Scaling of B_{sat} with neutronization rate $\dot{\mu}_0$
For parameters $\lambda = (48\pi)^2$, $\chi = 1$, $\dot{\mu}_0 = f\lambda^2/10$, we vary f linearly from 1/50 to 1/5 to verify our derived power law scaling for B_{sat}
These values are determined at the point where the time derivative of μ displays a sudden spike (see top right)





Example simulation (f = 1/10)
Top left: time evolution of volume-ave. μ and helicity (from Eq. 5). μ is built up due to forcing but decreases due to the growth of B; crossing point indicates start of saturation.
Top right: time derivative of μ, showing distinction between forcing-dominated growth and eventual saturation
Bottom right: 2D slice of B_x in the xy plane, towards the end of the simulation



Given the sub-10^{16} Gauss upper bound, linear chiral effects are insufficient to explain magnetar fields, but may be significant for pulsars. We expect non-linear effects to be less efficient, and they will be explored next.

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